COLLÈGE DE FRANCE
1530

Chaire de Physique
de la Matière Condensée
Antoine Georges

De l'effet Hall quantique aux matériaux moirés

- Topologie et géométrie des matériaux quantiques -

Cours 5 – *Effet Hall quantique fractionnaire (suite)*

Etats de bord. Fonctions de Wannier et obstructions topologiques

Cycle 2025-2026
27 mai 2026

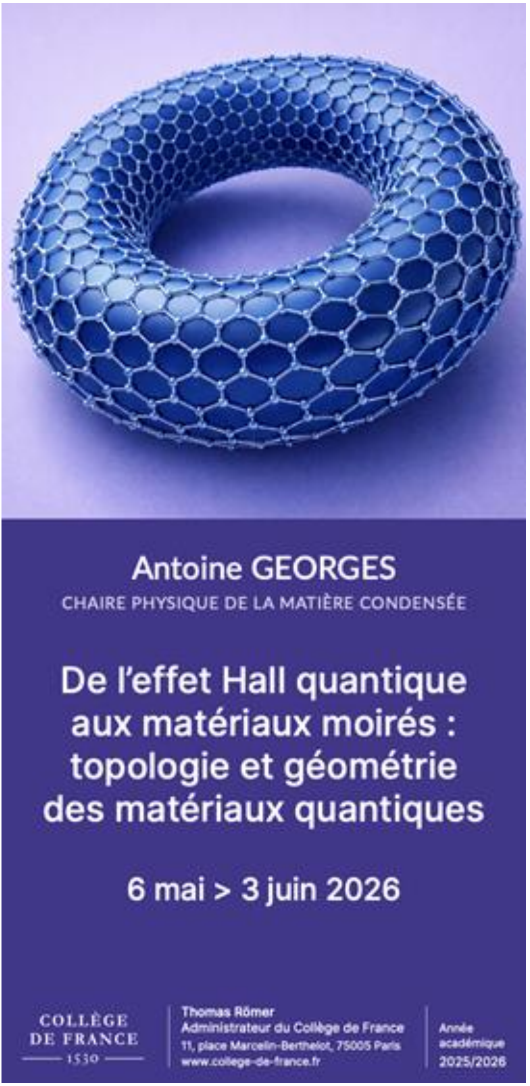
COLLÈGE DE FRANCE
1530

Chaire de Physique
de la Matière Condensée
Antoine Georges

From the Quantum Hall Effect to Moiré Materials
- Topology and Geometry of Quantum Materials -

Lecture 5 – The Fractional Quantum Hall Effect (cont'd)
Edge states. Wannier functions and topological obstructions

2025-2026 Lectures
May 27, 2026



Mercredi 27 mai 2026
Amphithéâtre Guillaume Budé

9h30 : Cours – Antoine Georges
Effet Hall quantique fractionnaire, suite.
Etats de bord, Fonctions de Wannier et obstructions topologiques

11h30 : Séminaire – Rebeca Ribeiro-Palau
(C2N, Université Paris-Saclay)
Topological states in moiré materials

Abstract: This seminar will present an experimental overview of several classes of topological states in 2D electronic systems. In the first part, I will discuss topological phases that emerge under strong magnetic fields, focusing on the integer and fractional quantum Hall effects. I will then address how these states are modified in moiré superlattices, where the interplay between electronic interactions, magnetic field effects, and band topology leads to new correlated and topological phases. In the second part, I will review topological states that arise without an external magnetic field through spontaneous breaking of time-reversal symmetry. In particular, I will discuss the anomalous Hall effect and recent observations of fractional anomalous Hall states. Together, these results highlight the richness of modern topological condensed matter physics.

Les cours seront enregistrés et diffusés sur le site web de la chaire après la date de la séance. Inscription sur la liste de diffusion : envoyer un message à listes-diffusion.cdf@college-de-france.fr avec comme sujet : subscribe chaire-pmc.ipcdf

Topological quantum matter: a conference in memory of Mark Oliver Goerbig

Orsay (France)
24-25 Sep 2026

Speakers

Emmanuel Baudin (LPENS Paris)

Lara Benfatto (Sapienza, University of Rome)

Silke Biermann (CPhT, Ecole Polytechnique)

David Carpentier (LPENS Lyon)

Marc Gabay (LPS Orsay)

Lih-King Lim (Zhejiang University, Hangzhou)

Xin Lu (ShanghaiTech University)

Leonardo Mazza (LPTMS Orsay)

Roderich Moessner (MPIPKS Dresden)

Gilles Montambaux (LPS Orsay)

Miguel Monteverde (LPS Orsay)

Cristiane Morais-Smith (Utrecht University)

Milan Orlita (LNCMI Grenoble)

Zlatko Papic (University of Leeds)

Nicolas Regnault (Flatiron Institute, New-York)

Florian Simon (TU München)

Kang Yang (Westlake University, Hangzhou)

https://markgoerbigconf.sciencesconf.org

Mailing List

(Weekly announcement of lecture and seminar, etc.)

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Subject line: subscribe chaire-pmc.ipcdf
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Website:

https://www.college-de-france.fr/site/antoine-georges/index.htm

Lectures are recorded
Videos are available on the CdF website and on YouTube
The current version of the notes on the website
is PRELIMINARY and contains (a few) mistakes/typos

Outline – Lecture 5

- FQHE, cont'd:
- Topological order
- Fractional statistics (exp:→ Seminar June 3)
- Neutral excitation: magnetoroton
- Edge states in topological systems and the bulk-boundary correspondence
- Wannier functions and topological obstructions

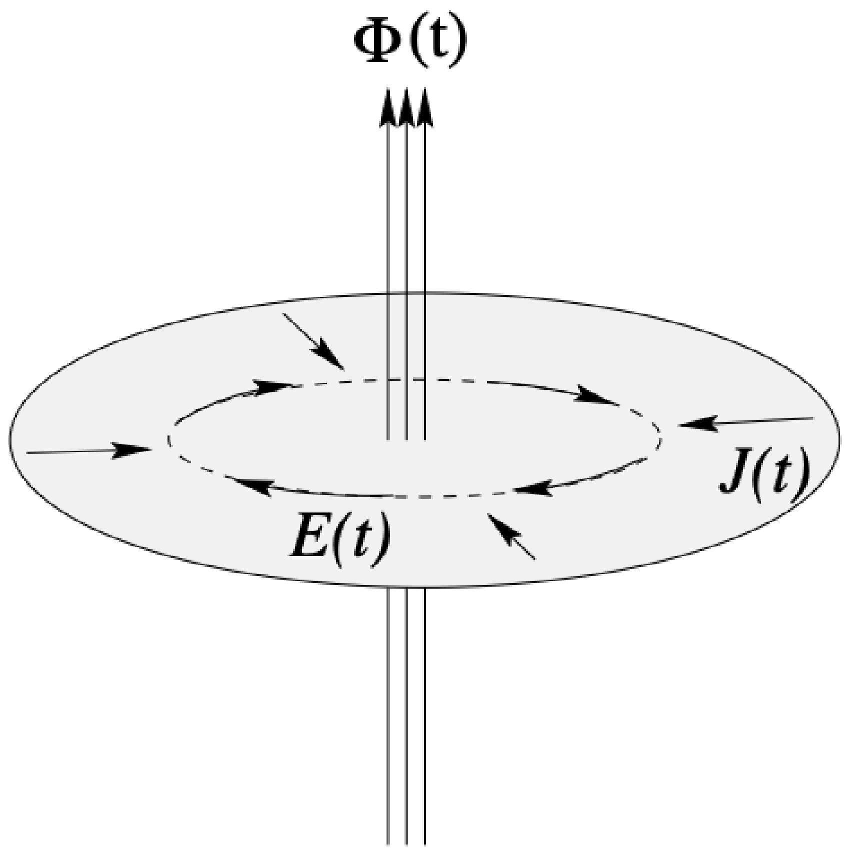
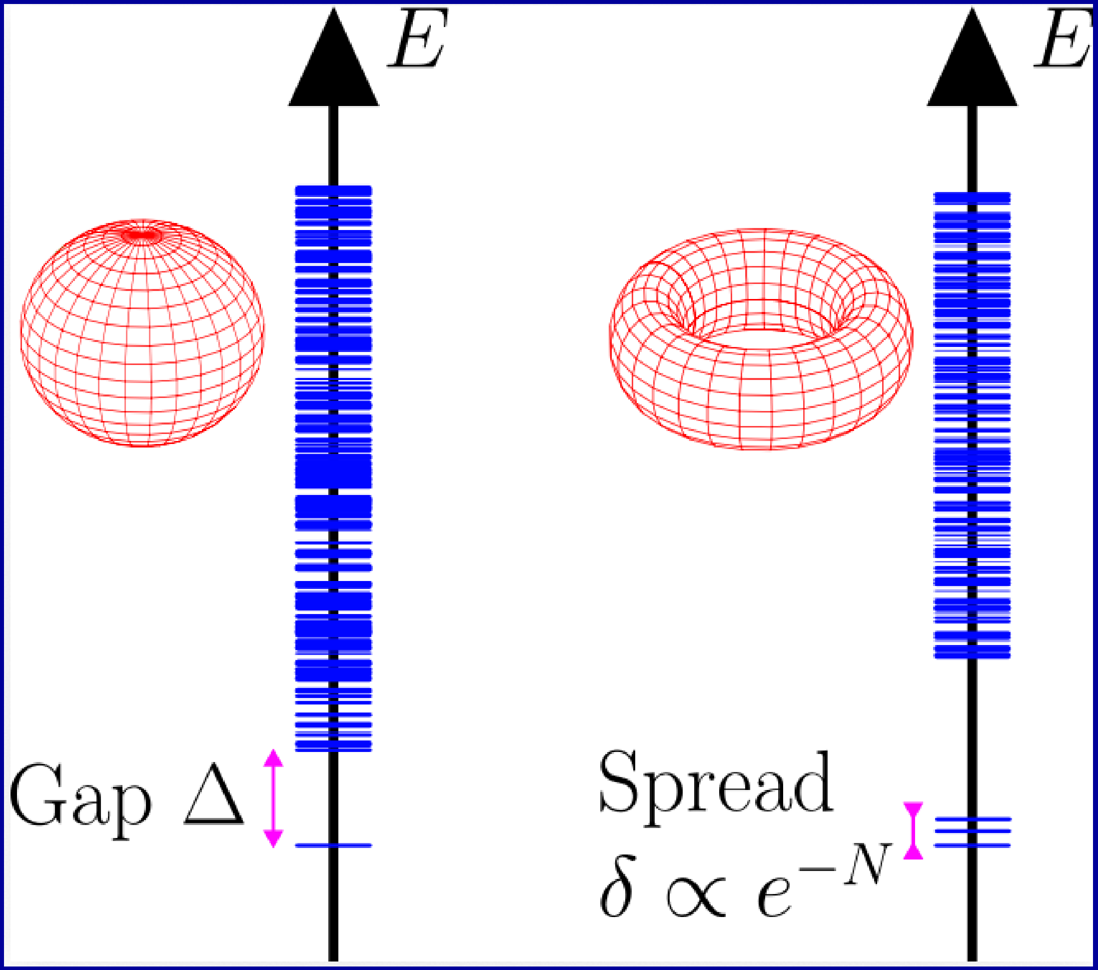


Figure 10: Construction of a Laughlin quasiparticle by adiabatically threading flux $\Phi(t)$ through a point in the sample. Faraday induction gives an azimuthal electric field $E(t)$ which in turn produces a radial current $J(t)$. For each quantum of flux added, charge νe flows into (or out of) the region due to the quantized Hall conductivity $\nu e^2/h$. A flux tube containing an integer number of flux quanta is invisible to the particles (since the Aharanov phase shift is an integer multiple of 2π) and so can be removed by a singular gauge transformation.

From S.Girvin, in Séminaire Poincaré

Topological order and degeneracy of the ground-state

No Degeneracy →



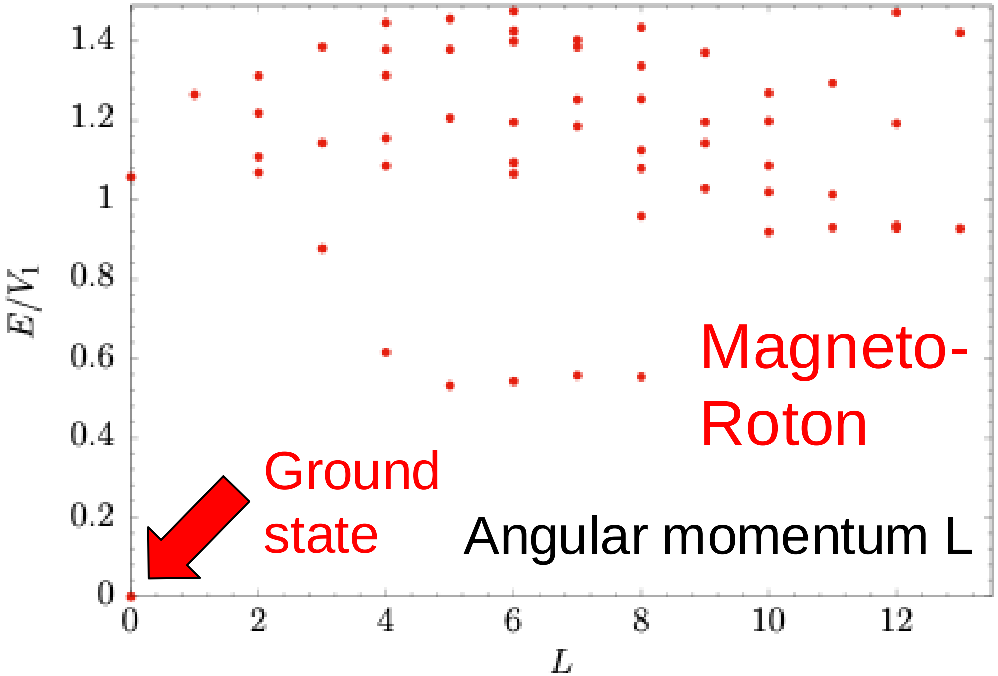
Ground-state degeneracy:
- In the infinite volume limit
- Exact at finite N with symmetries

← 3-fold

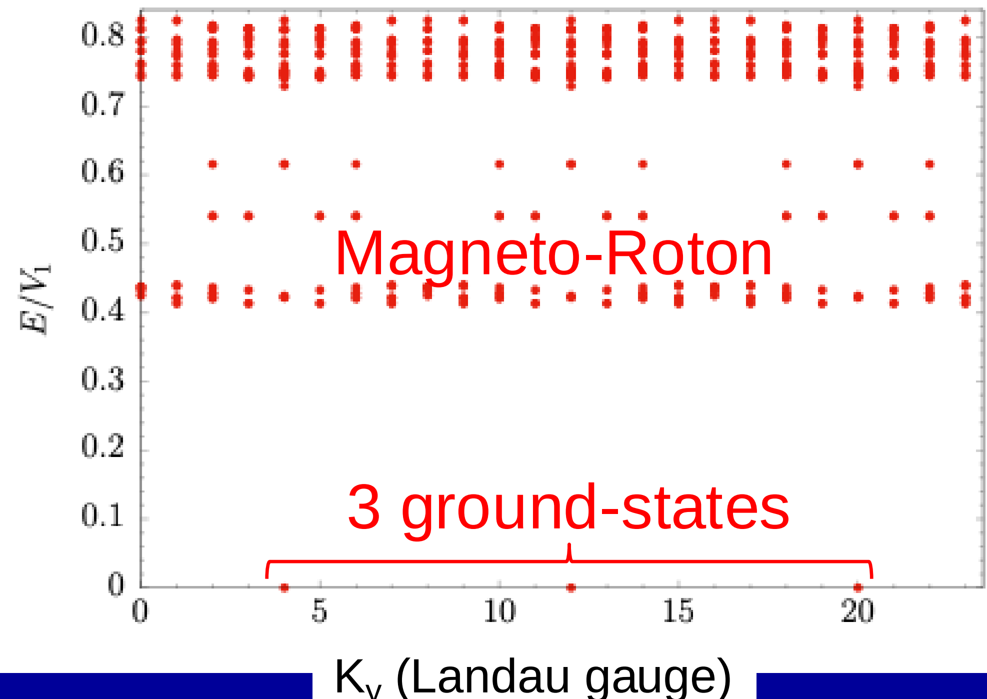
Figure: courtesy Nicolas Regnault

ED with 8 electrons
Haldane p.p.

← Sphere



Torus →



ED Results and Figures:
Courtesy Nicolas Regnault

Laughlin wave function on the torus with two inserted fluxes

$$\Psi_\alpha^{(\theta_1,\theta_2)}(z_1,\ldots,z_{N_e}) = \mathcal{N}\prod_{i<j}\vartheta_1\left(\frac{z_i-z_j}{L_1}\,\middle|\,\tau\right)^m \cdot \Theta_\alpha^{(\theta_1,\theta_2)}(Z_{\rm cm})\cdot\prod_j e^{-|z_j|^2/4\ell_B^2}$$

$$\Theta_\alpha^{(\theta_1,\theta_2)}(Z_{\rm cm}) = \vartheta\begin{bmatrix}\frac{\alpha+\theta_1/2\pi}{m}\\ \frac{\theta_2}{2\pi}\end{bmatrix}\left(\frac{m\,Z_{\rm cm}}{L_1}\,\middle|\,m\tau\right)$$

$\alpha = 0,\cdots,m-1$

labels the m ground-states

Jacobi theta-functions:

$$\vartheta\begin{bmatrix}a\\ b\end{bmatrix}(u|\tau) = \sum_{n\in\mathbb{Z}} e^{i\pi(n+a)^2\tau}e^{2\pi i(n+a)(u+b)}$$

$$\vartheta_1(u|\tau) = -\vartheta\begin{bmatrix}1/2\\ 1/2\end{bmatrix}(u|\tau)$$

Experimental evidence for fractional charge: shot noise

Direct observation of a fractional charge

R. de-Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin & D. Mahalu

Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

NATURE | VOL 389 | 11 SEPTEMBER 1997

See also Reznikov et al. Nature 399, 238 (1999)

C.Glattli, M.Reznikov: Europhysics Prize 1999

VOLUME 79, NUMBER 13 PHYSICAL REVIEW LETTERS 29 SEPTEMBER 1997

Observation of the $e/3$ Fractionally Charged Laughlin Quasiparticle

L. Saminadayar and D.C. Glattli

Service de Physique de l'État Condensé, CEA/Saclay, F-91191 Gif-sur-Yvette Cedex, France

Y. Jin and B. Etienne

Laboratoire de Microstructures et Microélectronique, CNRS, B.P. 107, F-92225 Bagneux Cedex, France

(Received 30 June 1997)

The existence of fractional charges carrying current is experimentally demonstrated. Using a 2D electron system in a high perpendicular magnetic field we measure the shot noise associated with tunneling in the fractional quantum Hall regime at Landau level filling factor $1/3$. The noise gives a direct determination of the quasiparticle charge, which is found to be $e^* = e/3$ as predicted by Laughlin. The existence of $e/3$ Laughlin quasiparticles is unambiguously confirmed by the shot noise to Johnson-Nyquist noise crossover found for temperature $\Theta = e^* V_{ds}/2k_B$. [S0031-9007(97)04194-X]

See Lecture 4 for more details

Experimental evidence for fractional statistics → seminar on June 3 by Gwendal Fève

MESOSCOPIC PHYSICS

Bartolomei *et al.*, *Science* **368**, 173–177 (2020) 10 April 2020

Fractional statistics in anyon collisions

H. Bartolomei[1]*, M. Kumar[1]*†, R. Bisognin[1], A. Marguerite[1]‡, J.-M. Berroir[1], E. Bocquillon[1], B. Plaçais[1], A. Cavanna[2], Q. Dong[2], U. Gennser[2], Y. Jin[2], G. Fève[1]§

Two-dimensional systems can host exotic particles called anyons whose quantum statistics are neither bosonic nor fermionic. For example, the elementary excitations of the fractional quantum Hall effect at filling factor $\nu = 1/m$ (where m is an odd integer) have been predicted to obey Abelian fractional statistics, with a phase ϕ associated with the exchange of two particles equal to π/m. However, despite numerous experimental attempts, clear signatures of fractional statistics have remained elusive. We experimentally demonstrate Abelian fractional statistics at filling factor $\nu = ⅓$ by measuring the current correlations resulting from the collision between anyons at a beamsplitter. By analyzing their dependence on the anyon current impinging on the splitter and comparing with recent theoretical models, we extract $\phi = \pi/3$, in agreement with predictions.

APS Buckley Prize 2026
To Gwendal Fève and Michael Manfra

nature physics

NATURE PHYSICS | VOL 16 | SEPTEMBER 2020 | 931–936 |

ARTICLES

https://doi.org/10.1038/s41567-020-1019-1

Check for updates

Direct observation of anyonic braiding statistics

J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] and M. J. Manfra[1,2,3,4,5] ✉

Anyons are quasiparticles that, unlike fermions and bosons, show fractional statistics when two of them are exchanged. Here, we report the experimental observation of anyonic braiding statistics for the $\nu = 1/3$ fractional quantum Hall state by using an electronic Fabry–Perot interferometer. Strong Aharonov–Bohm interference of the edge mode is punctuated by discrete phase slips that indicate an anyonic phase $\theta_{anyon} = 2\pi/3$. Our results are consistent with a recent theory that describes an interferometer operated in a regime in which device charging energy is small compared to the energy of formation of charged quasiparticles, which indicates that we have observed anyonic braiding.

Neutral Excitation: The `Magneto-Roton’

VOLUME 54, NUMBER 6 PHYSICAL REVIEW LETTERS 11 FEBRUARY 1985

Collective-Excitation Gap in the Fractional Quantum Hall Effect

S. M. Girvin
Surface Science Division, National Bureau of Standards, Gaithersburg, Maryland 20899

and

A. H. MacDonald
National Research Council of Canada, Ottawa, K1A OR6, Canada

and

P. M. Platzman
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 25 October 1984)

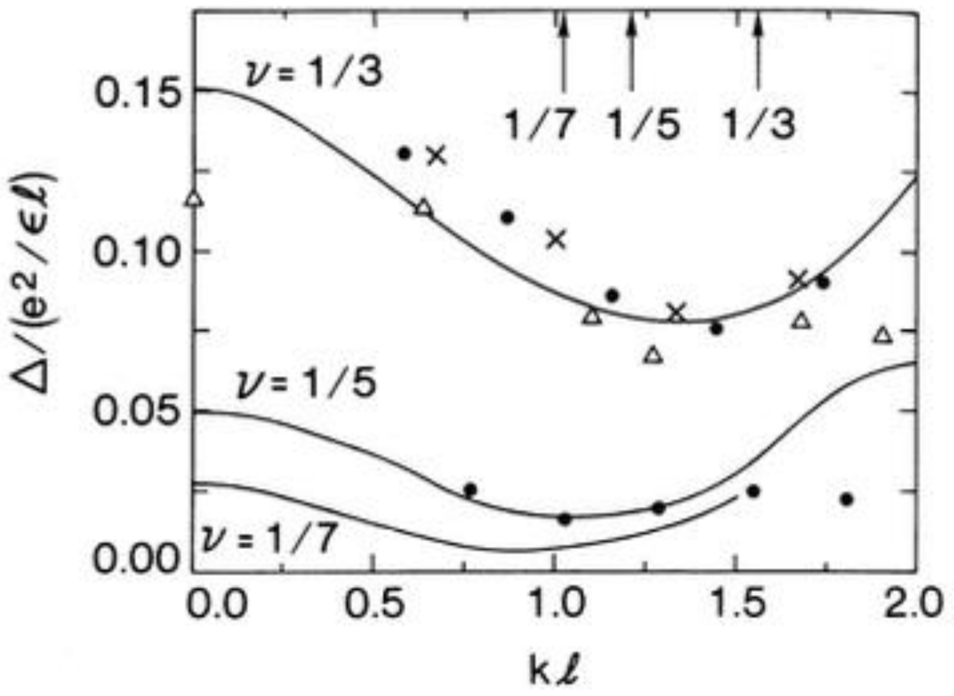
We present a theory of the collective excitation spectrum in the fractional quantum Hall-effect regimes, in analogy with Feynman's theory for helium. The spectrum is in excellent quantitative agreement with the numerical results of Haldane. *Within this approximation* we prove that a finite gap is generic to any liquid state in the extreme quantum limit and that in this single-mode *approximation* gapless excitations can arise only as Goldstone modes for ground states with broken translation symmetry.

PHYSICAL REVIEW B VOLUME 33, NUMBER 4 15 FEBRUARY 1986

Magneto-roton theory of collective excitations in the fractional quantum Hall effect

Variational wave-function:

$$\Psi_{\mathbf{k}}(\{r_i\}) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} e^{i\mathbf{k}\cdot\mathbf{r}_i}\, \Psi_L(\{r_i\})$$



Note: no backflow!

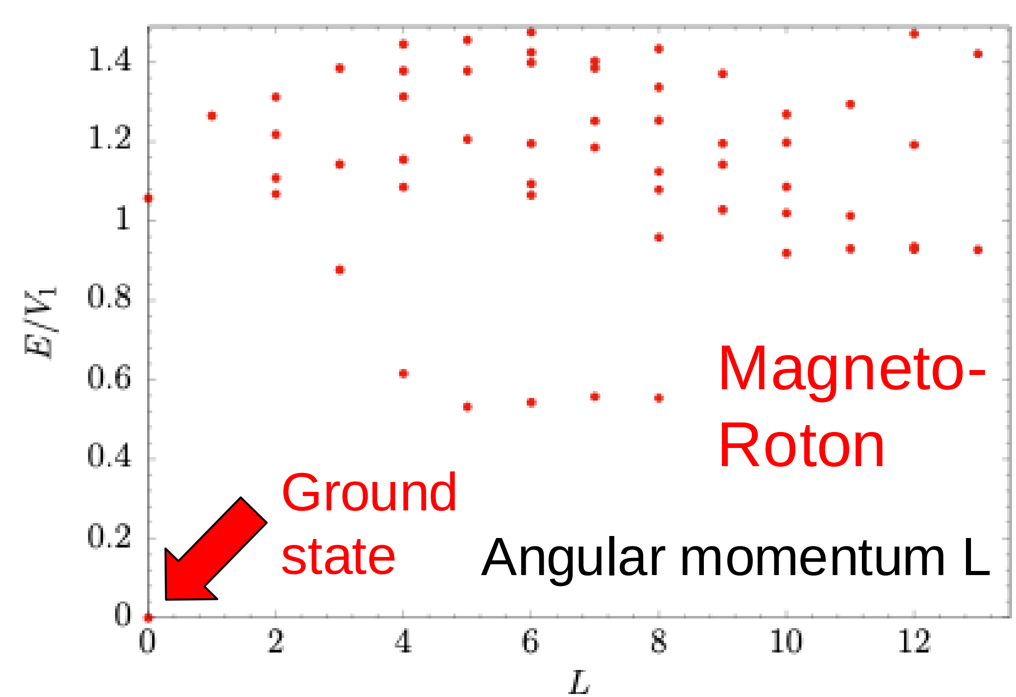
Roton Minimum: Hint of Wigner crystallisation

S.Girvin Séminaire Poincaré

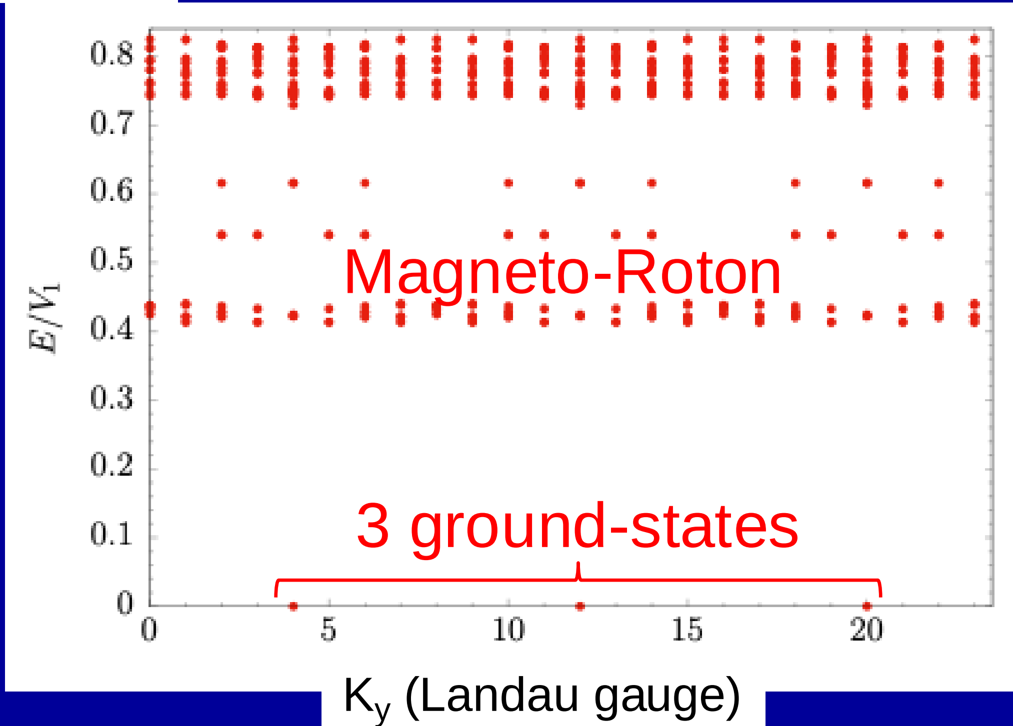
Figure 8: Comparison of the single mode approximation (SMA) prediction of the collective mode energy for filling factors $\nu = 1/3, 1/5, 1/7$ (solid lines) with small-system numerical results for N particles. Crosses indicate the $N = 7, \nu = 1/3$ spherical system, triangles indicate the $N = 6, \nu = 1/3$ hexagonal unit cell system results of Haldane and Rezayi [18]. Solid dots are for $N = 9, \nu = 1/3$ and $N = 7, \nu = 1/5$ spherical system calculations of Fano et al. [19] Arrows at the top indicate the magnitude of the reciprocal lattice vector of the Wigner crystal at the corresponding filling factor. Notice that unlike the phonon collective mode in superfluid helium shown in Fig. (7), the mode here is gapped.

ED with 8 electrons
Haldane p.p.

← Sphere



Torus →



ED Results and Figures:
Courtesy Nicolas Regnault

Magneto-roton states from ED

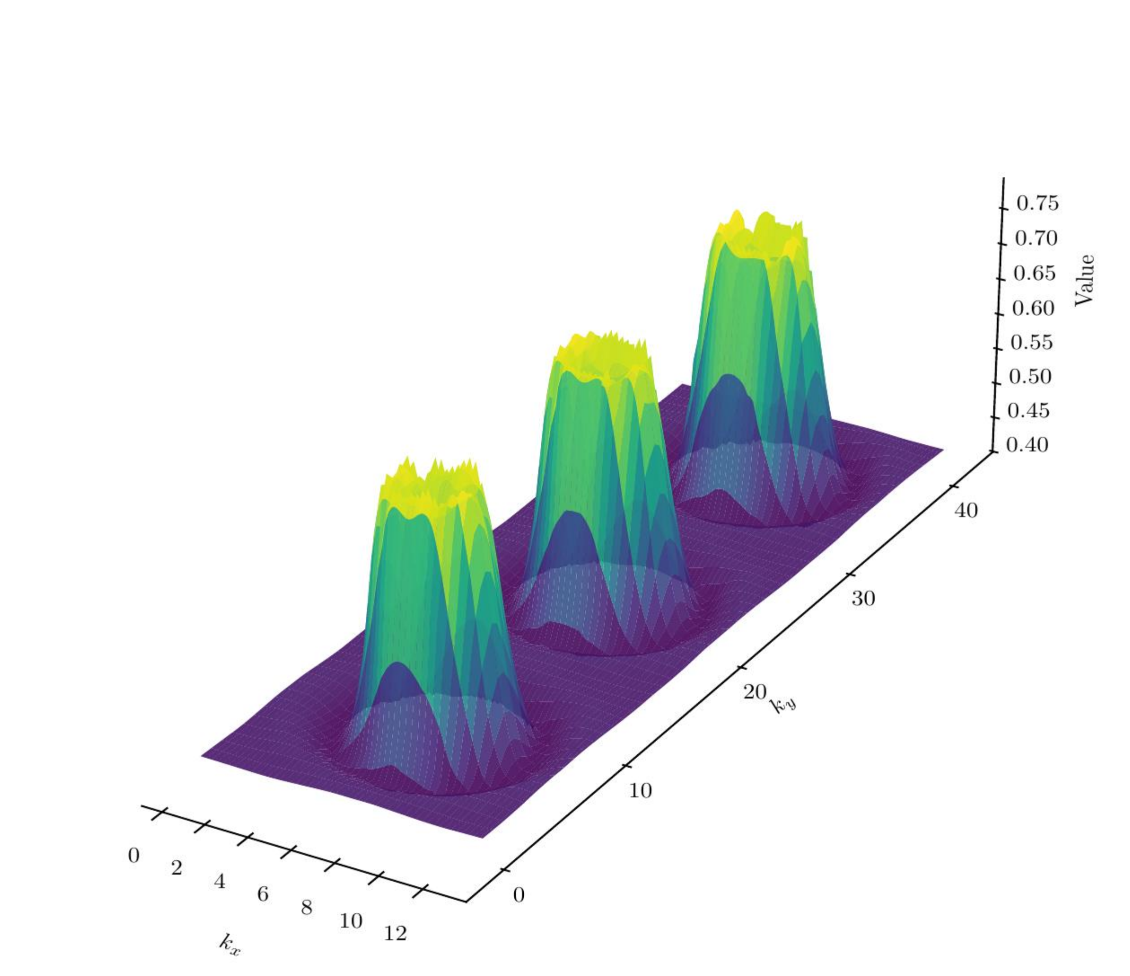


Figure: courtesy Nicolas Regnault

First Experimental Observation: Raman Scattering

VOLUME 70, NUMBER 25 PHYSICAL REVIEW LETTERS 21 JUNE 1993

Observation of Collective Excitations in the Fractional Quantum Hall Effect

A. Pinczuk, B. S. Dennis, L. N. Pfeiffer, and K. West

AT&T Bell Laboratories, Murray Hill, New Jersey 07974

(Received 27 January 1993)

A long wavelength, low-energy excitation of the fractional quantum Hall state at $\nu=\frac{1}{3}$ has been observed by inelastic light scattering. The mode appears as a very sharp peak with marked temperature and magnetic field dependence. Its energy is consistent with theoretical predictions for the collective gap excitations of the incompressible quantum fluid. Spectra interpreted as $q=0$ collective spin-wave excitations also display the strong dependence on field and temperature associated with the fractional quantum Hall state.

PACS numbers: 73.40.Hm, 73.20.Dx, 73.20.Mf, 78.30.Fs

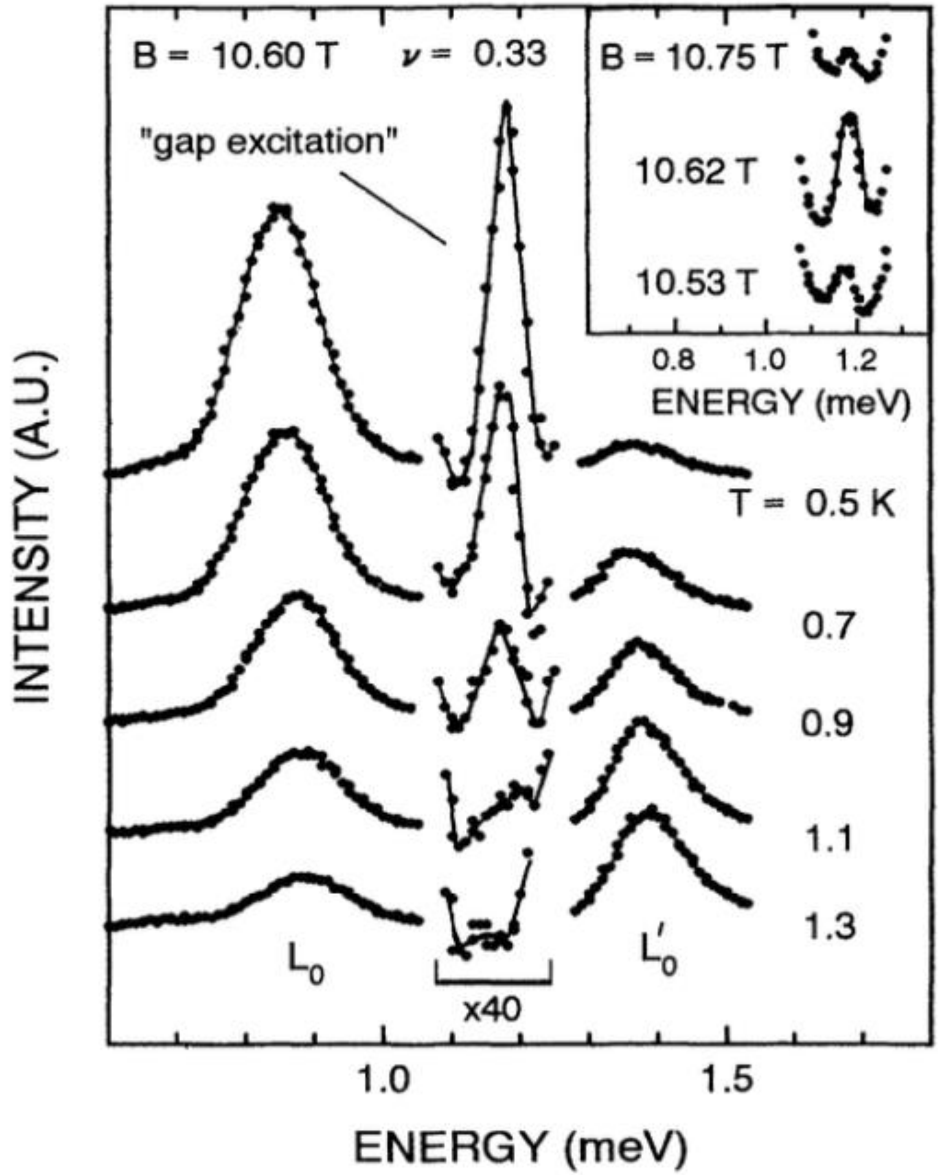


FIG. 1. Temperature dependence of inelastic light scattering spectra of a low-lying excitation of the FQHE at $\nu=\frac{1}{3}$. The single quantum well has density $n=8.5\times10^{10}$ cm^{-2}. The inset shows the B dependence of the 0.5 K spectra. The light scattering peak, labeled "gap excitation," is interpreted as a $q=0$ collective gap excitation. The bands labeled L_0 and L_0' comprise the characteristic doublets of intrinsic photoluminescence. The temperature dependence of the L_0 and L_0' intensities is due to the optical anomaly at $\nu=\frac{1}{3}$.

Momentum Dependence
(Surface Acoustic Waves)

Dispersion of the Excitations of Fractional Quantum Hall States

Igor V. Kukushkin,[1,2] Jurgen H. Smet,[1]* Vito W. Scarola,[3,4]
Vladimir Umansky,[5] Klaus von Klitzing[1]

22 MAY 2009 VOL 324 **SCIENCE**

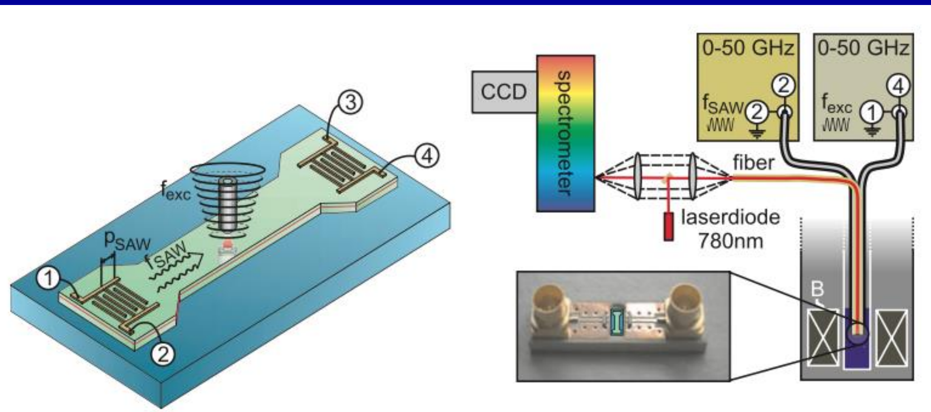
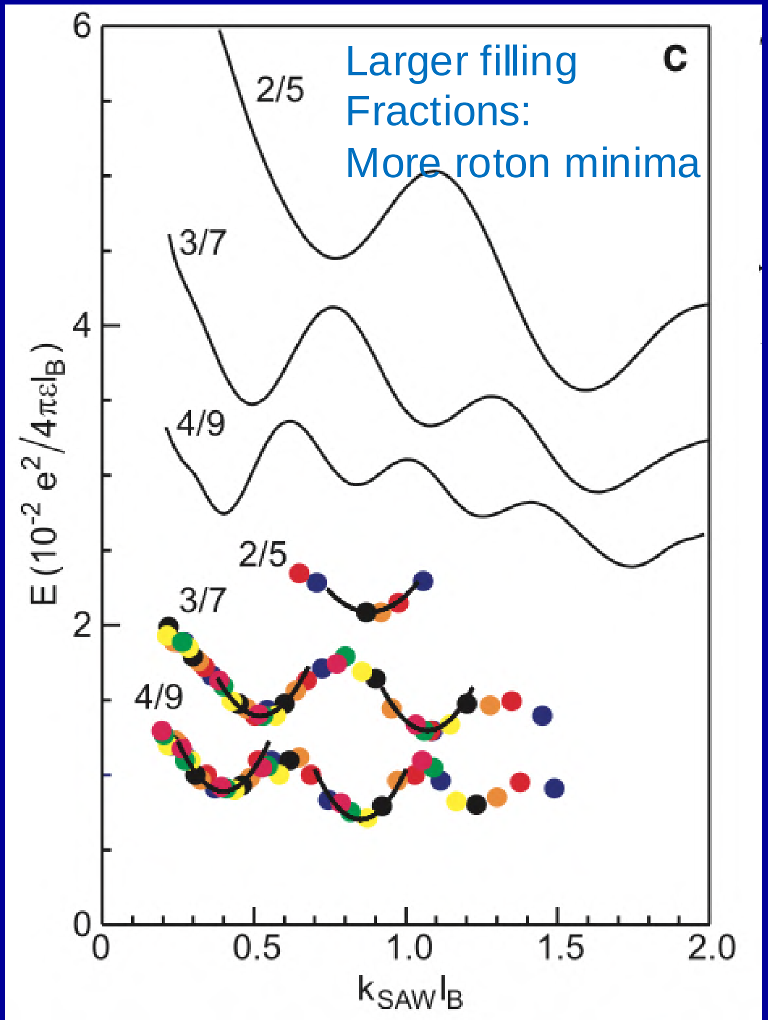


Fig. 1. Experimental arrangement for the detection of resonant microwave absorption at large wave vectors. (Left) Sample geometry consisting of a 0.1-mm-wide and 1-mm-long mesa. At its ends, the mesa widens and hosts two interdigital transducers with period p_{SAW}. High-frequency radiation drives the left transducer. The transducer launches SAWs across the sample. In the active-device region, light from a 780-nm laserdiode triggers a luminescence signal. This region of the sample is also irradiated with a quasi-monochromatic microwave by using a second high-frequency generator. Electrodes 1 and 4, which belong to transducers on opposite sides of the mesa, serve as a dipole antenna. (Right) Schematic of the cryostat configuration and the high-frequency chip carrier.



Graviton-like modes?

Article

Evidence for chiral graviton modes in fractional quantum Hall liquids

https://doi.org/10.1038/s41586-024-07201-w
Received: 12 March 2023
Accepted: 16 February 2024

Jiehui Liang[1,7], Ziyu Liu[2,7], Zihao Yang[1], Yuelei Huang[1], Ursula Wurstbauer[3], Cory R. Dean[2], Ken W. West[4], Loren N. Pfeiffer[4], Lingjie Du[1,5]✉ & Aron Pinczuk[2,6]

Today in the context of Marc Henneaux's lectures:

27
MAI
2026

16:00 à 17:30

SÉMINAIRE

Non-Relativistic Limits of Massive (Higher-Spin) gravity and Their Condensed Matter Applications

Andrea Campoleoni

Limites non relativistes de la théorie d'Einstein et applications

The Bulk-Boundary Correspondence. Edge States

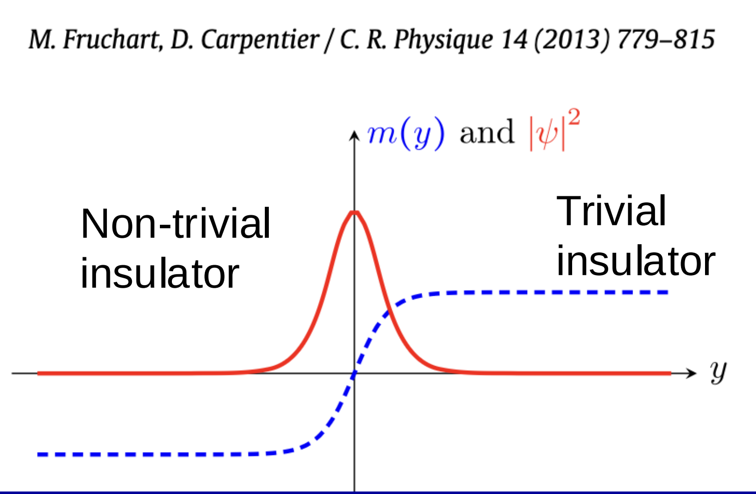
Edge States – Chern Insulator

Consider an interface at y=0 between a trivial insulator and a topological CI with C=+1: the `mass' changes sign at y=0 and the gap closes and reopens.

$$[m(y)\sigma_z - i\sigma_x\partial_x - i\sigma_y\partial_y]\,\psi(x,y) = E\psi(x,y)$$

$$\boxed{\psi(x,y) \propto e^{iqx}\, e^{-\int_0^y m(y')\,dy'}\, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

$v_F=1$

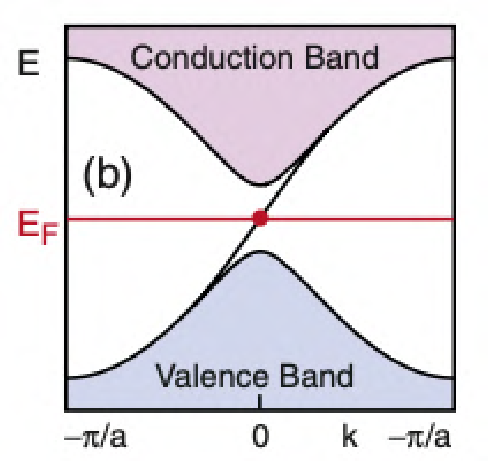


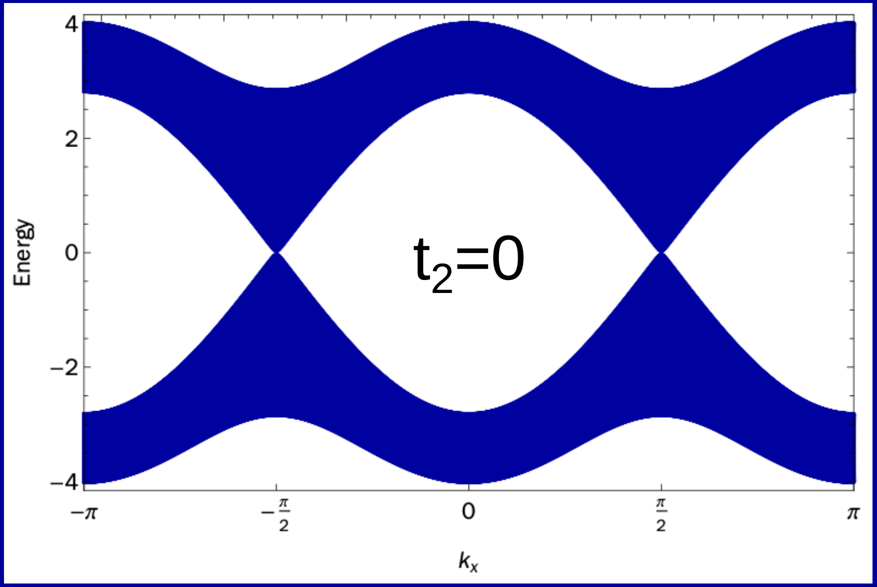
cf. Jackiw and Rebbi, Phys Rev D 13, 3398 (1976)

Su, Schriffer and Heeger (SSH model, 1979)

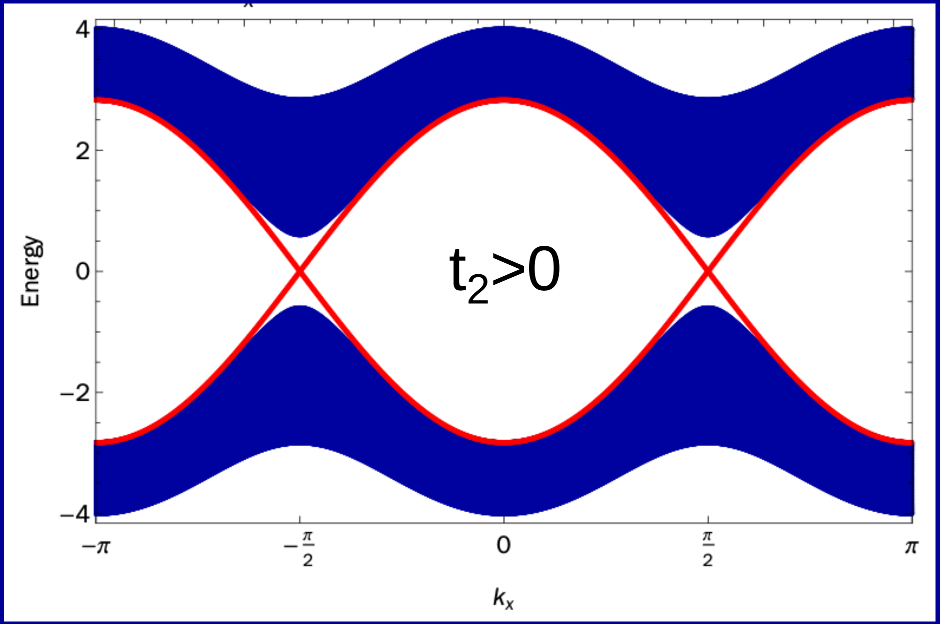
Diagonalisation on a strip of finite width

(Square lattice Haldane model, width 200, m=0)





Diagonalization of $H(k_x,y)$ mixed representation



Slab of Haldane insulator between → two interfaces

Why edge states?

- Across the interface, we expect the wave-functions to deform continuously from those in material 1 to material 2
- But between a topological and trivial insulator, this is not possible: hence the gap must close at the interface, leading to zero-energy states

How many edge states?
Index Theorem
Bulk-Boundary Correspondence

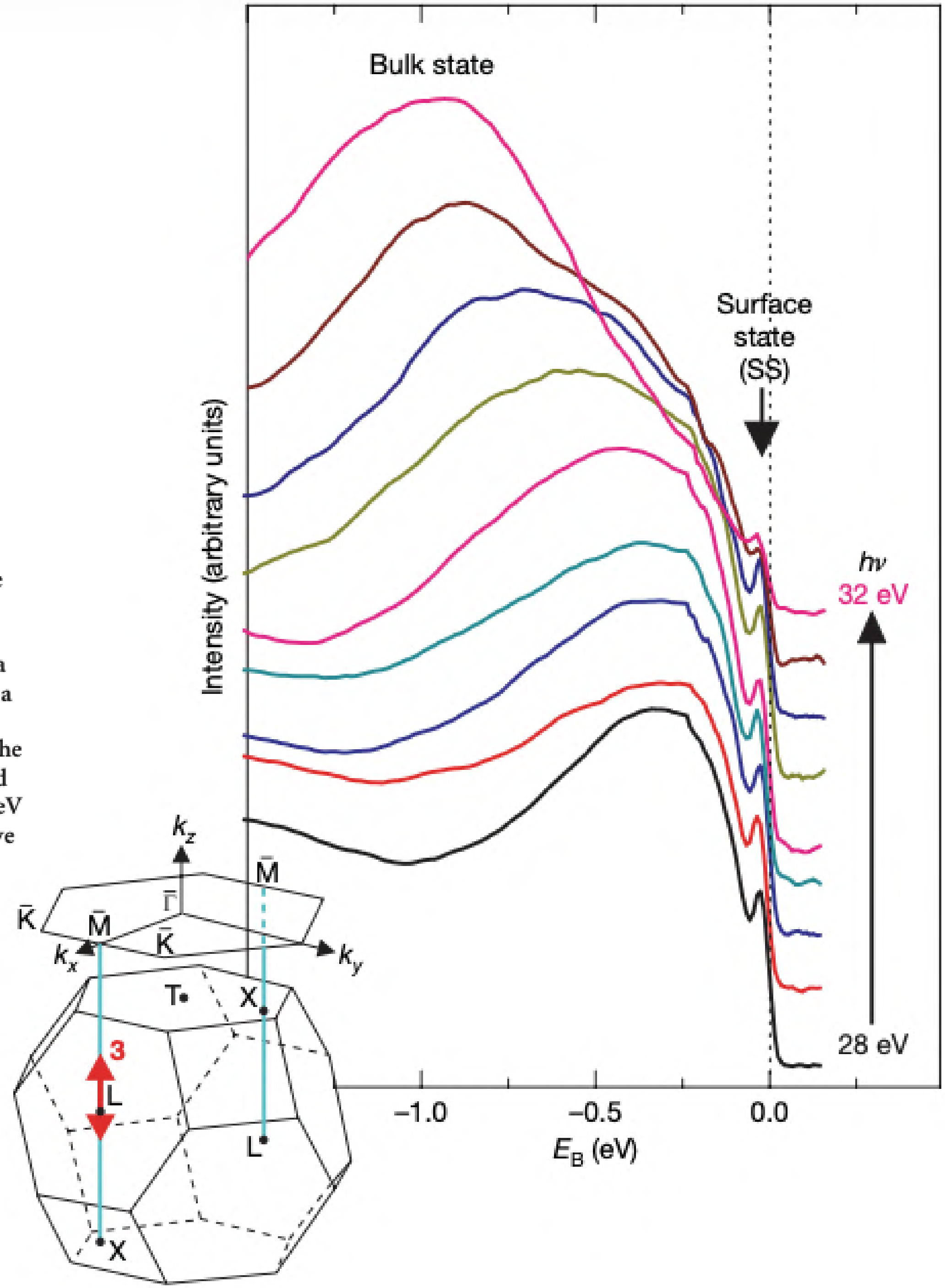
The difference between the number of right-moving and left-moving edge modes is equal to the difference of the bulk Chern number across the interface:

$$N_R - N_L = \Delta C$$

This is a special case of the *Atiyah-Singer theorem*, which relates the difference between the number of zero modes of an elliptic operator on a compact manifold to a topological index

Observation of edge states with ARPES in a 3D topological insulator: $Bi_{1-x}Sb_x$

Figure 2 | Dispersion along the cut in the k_z-direction. Surface states are experimentally identified by studying their out-of-plane momentum dispersion through the systematic variation of incident photon energy. **a**, EDCs of $Bi_{0.9}Sb_{0.1}$ with electrons at the Fermi level (E_F) maintained at a fixed in-plane momentum of ($k_x = 0.8$ Å^{-1}, $k_y = 0.0$ Å^{-1}) are obtained as a function of incident photon energy to identify states that exhibit no dispersion perpendicular to the (111) plane along the direction shown by the double-headed arrow labelled '3' in the inset (see Methods for the detailed procedure). Selected EDC data sets with photon energies ranging from 28 eV to 32 eV in steps of 0.5 eV are shown for clarity. The non-energy-dispersive (k_z-independent) peaks near E_F are the surface states. **b**, ARPES intensity



Hsieh et al. (MZ Hasan's group)
Nature 452, 970 (2008)

Edge states in the IQHE

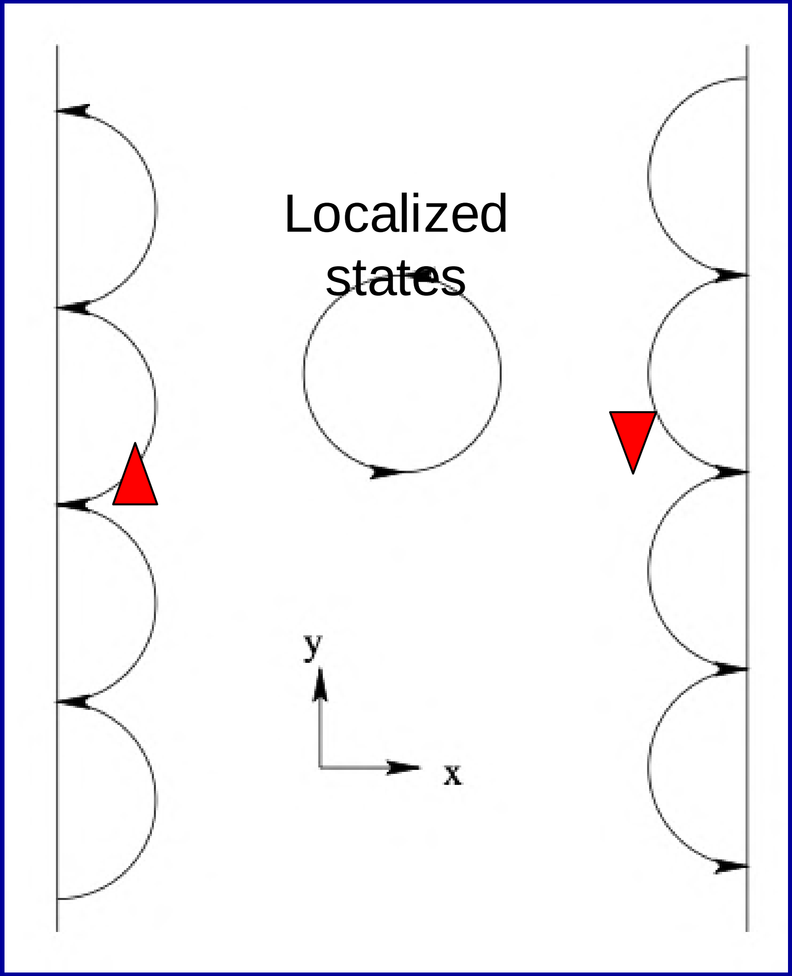
PHYSICAL REVIEW B VOLUME 25, NUMBER 4 15 FEBRUARY 1982

Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential

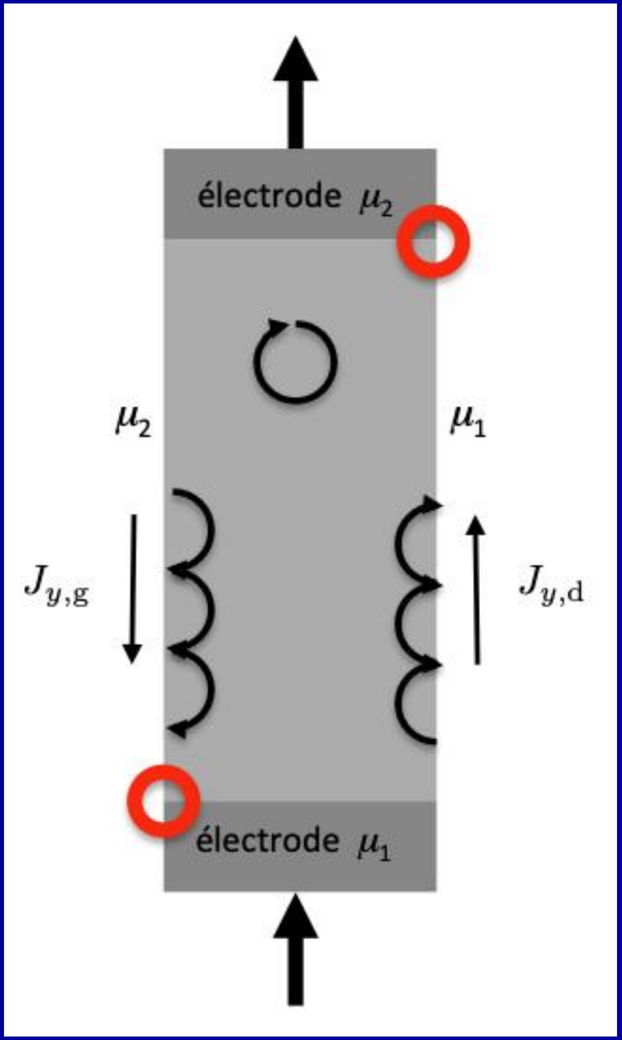
B. I. Halperin
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 21 August 1981)

When a conducting layer is placed in a strong perpendicular magnetic field, there exist current-carrying electron states which are localized within approximately a cyclotron radius of the sample boundary but are extended around the perimeter of the sample. It is shown that these quasi-one-dimensional states remain extended and carry a current even in the presence of a moderate amount of disorder. The role of the edge states in the quantized Hall conductance is discussed in the context of the general explanation of Laughlin. An extension of Laughlin's analysis is also used to investigate the existence of extended states in a weakly disordered two-dimensional system, when a strong magnetic field is present.



Semi-classical picture
of QHE chiral edge states:
skipping orbits
(after Grivin, 1999)



Two terminal measurement
(after M.O. Goerbig and J.Dalibard)

IQHE Edge states have transmission t=1
In the Landauer sense (no backscattering)
corresponding to the integer quantization
of the Hall conductance

Observing edge states with local probes


Article

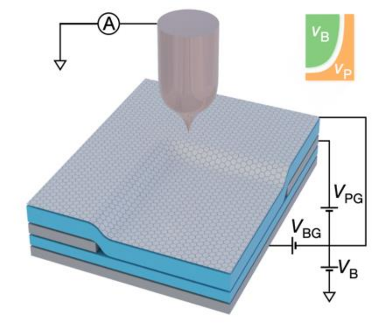
Visualizing interaction-driven restructuring of quantum Hall edge states

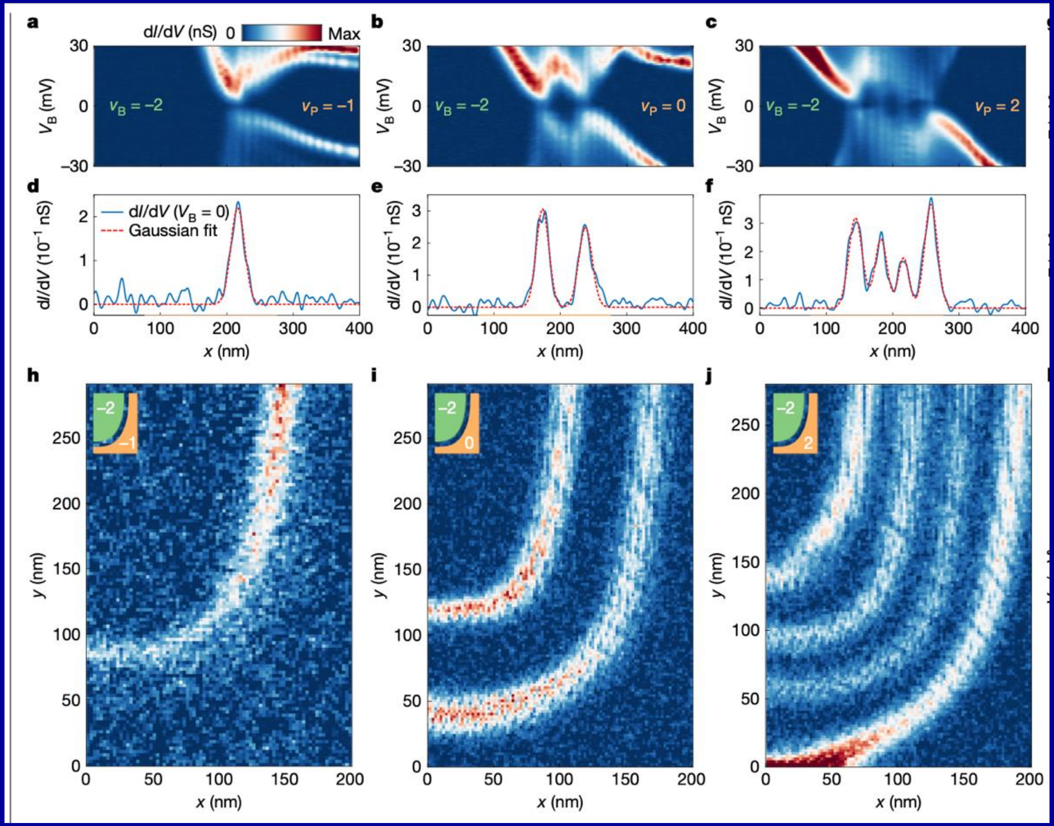
https://doi.org/10.1038/s41586-025-09858-3

Received: 30 June 2025

Accepted: 4 November 2025

Jiachen Yu[1,2,7], Haotan Han[1,2,7], Kristina G. Wolinski[1,2,7], Ruihua Fan[3], Amir S. Mohammadi[3], Tianle Wang[3], Taige Wang[3], Liam Cohen[4], Kenji Watanabe[5], Takashi Taniguchi[6], Andrea F. Young[4], Michael P. Zaletel[3] & Ali Yazdani[1,2]✉

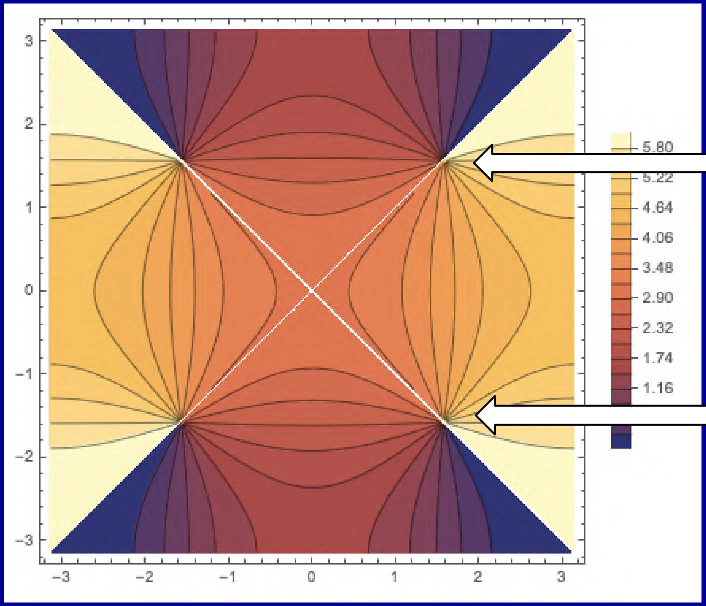




Wannier Functions: Topological Obstructions - See notes-

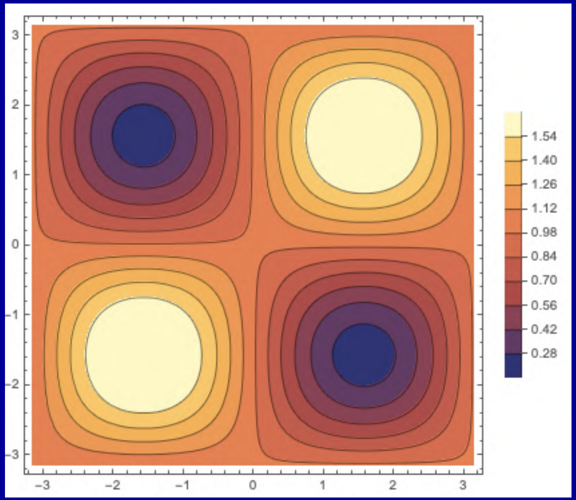
Square Haldane Model - Reminder

Phase Φ_k in BZ:



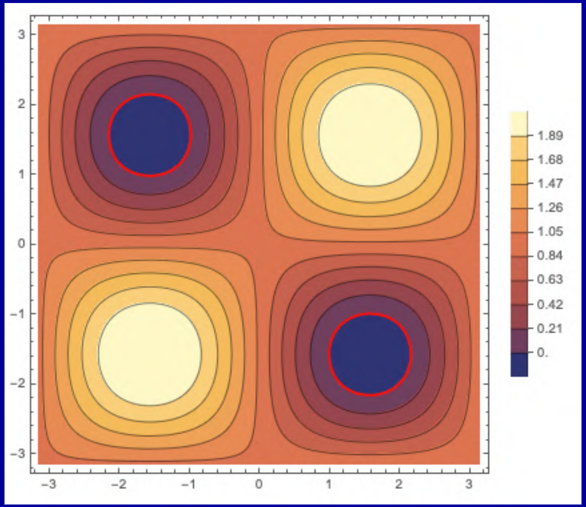
vortex

anti-vortex



Trivial phase: only one pole is reached

Topological phase: both poles are reached



Map of d_z:

Example: Wannier functions in a transition-metal oxide

PHYSICAL REVIEW B **74**, 125120 (2006)

Dynamical mean-field theory using Wannier functions: A flexible route to electronic structure calculations of strongly correlated materials

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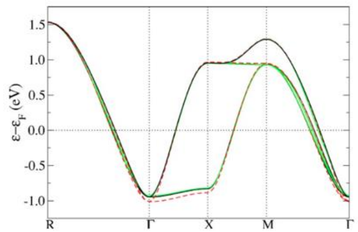
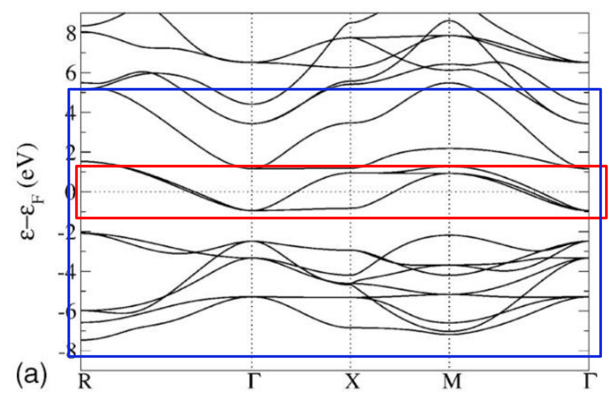
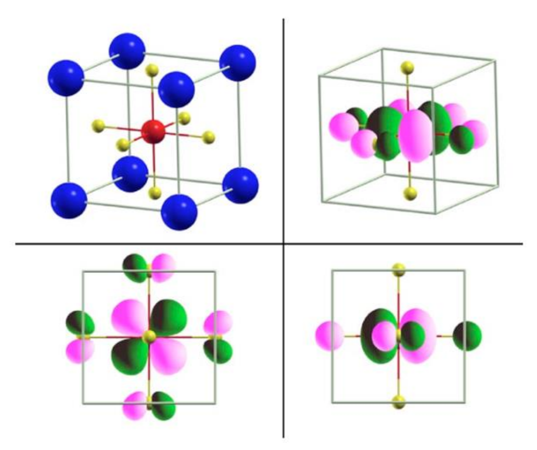


FIG. 3. (Color online) t_{2g} bands for $SrVO_3$ using different schemes to compute the t_{2g} Wannier functions (and the underlying LDA band structure or potential). Dark, MLWF(MBPP); dashed-red (dashed-gray), MLWF(FLAPW); and green (light gray), NMTO(LMTO-ASA). The t_{2g} bandwidth is marginally larger in FLAPW, leading to small differences.





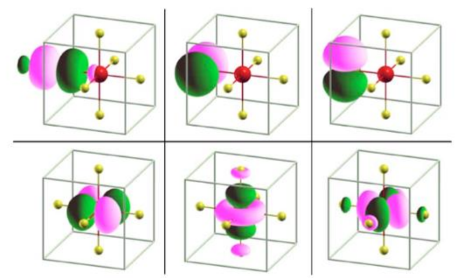


FIG. 7. (Color online) Distinct WFs for $SrVO_3$ obtained from the MLWF construction using the MBPP code. First row: O(p_x), O(p_y), and O(p_z) for a chosen oxygen site. Second row: V(t_{2g},xy) as well as V(e_g,$3z^2-r^2$) and V(e_g,x^2-y^2). The contour value for each of the MLWFs was chosen as 0.05 (a.u.)$^{-3/2}$.

Example: Wannier orbitals for t-WSe_2

PHYSICAL REVIEW B **113**, L081106 (2026)

Letter

Site-polarized Mott phases competing with a correlated metal in twisted WSe_2

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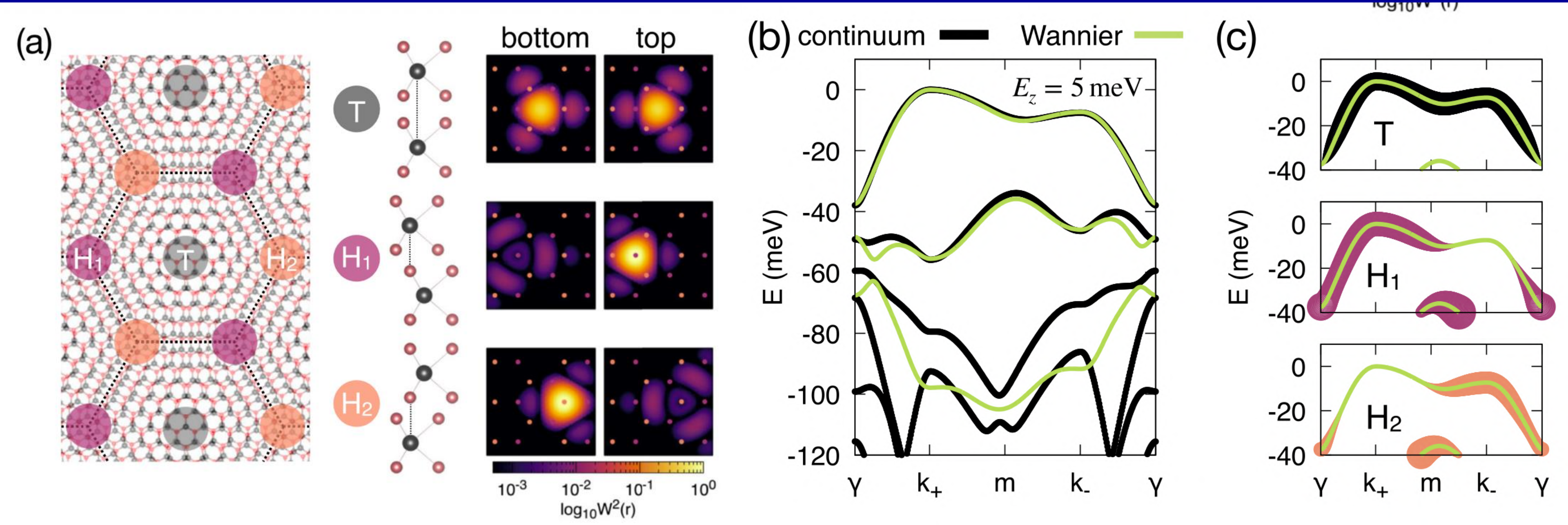

"Fragile" Topology

PHYSICAL REVIEW LETTERS **121,** 126402 (2018)

Editors' Suggestion

Fragile Topology and Wannier Obstructions

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Application to TBLG:

PHYSICAL REVIEW B **99**, 195455 (2019)

Editors' Suggestion

Faithful tight-binding models and fragile topology of magic-angle bilayer graphene

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