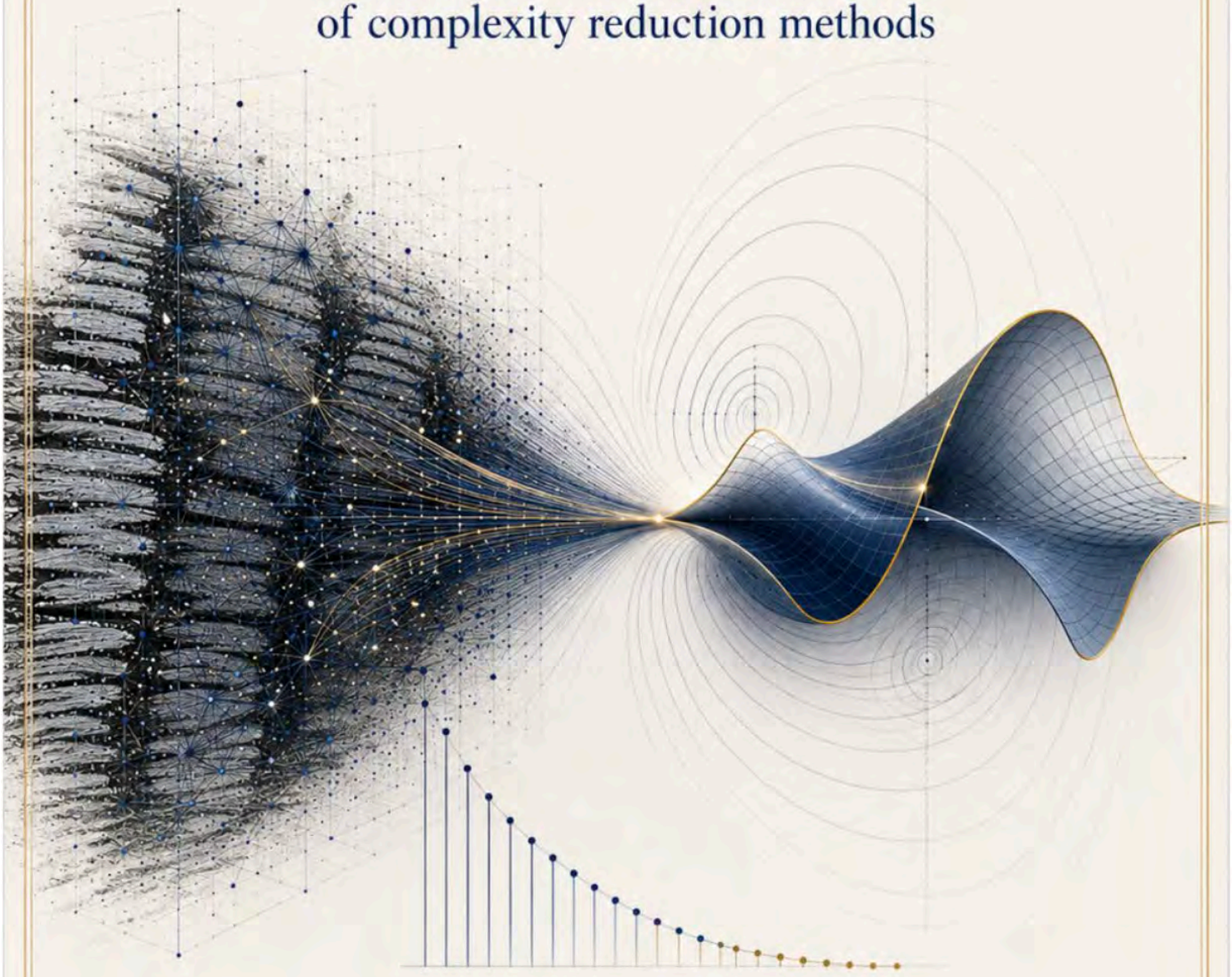
Book of Abstracts

Mathematical and applied aspects of complexity reduction methods



Symposium organized within the Annual Chair of Yvon Maday

Computer Sciences and Digital Technologies / Informatique et sciences numériques

COLLÈGE DE FRANCE
1530

22–24 June 2026

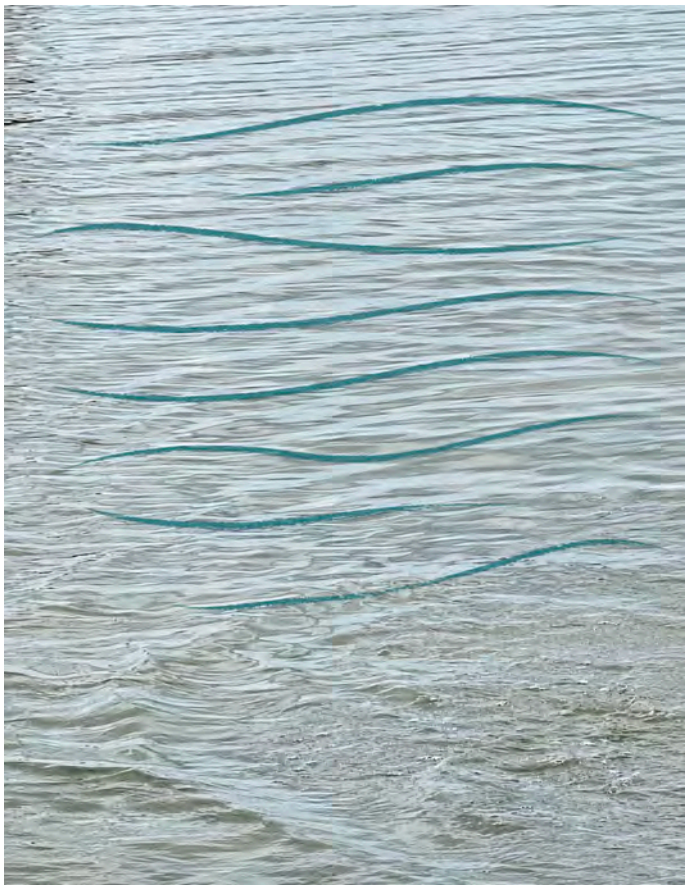
Collège de France, Paris

Salle 5, Site Marcelin Berthelot

SORBONNE UNIVERSITÉ
CRÉATEURS DE FUTURS DEPUIS 1257

Conference Framework

Mathematical and applied aspects of complexity reduction methods



Symposium within the Annual Chair
Computer Sciences and Digital Technologies

At Collège de France in partnership with Inria
of
Yvon Maday

June 22-23-24, 2026
Collège de France, Paris
Amphithéâtre Delmas-Marty (salle 5), Site Marcelin Berthelot

This symposium is organized with the support and participation of Sorbonne Université.

International Symposium

Mathematical and applied aspects of complexity reduction methods

This international symposium concludes the Annual Chair by broadening the perspective developed throughout the lectures and seminars devoted to complexity reduction methods. The lectures provided a mathematical overview of reduced basis and related model reduction approaches, including their foundations, a priori and a posteriori error estimation, computational implementation, recent nonlinear extensions, and the increasing role of learning-based techniques.

The subsequent seminar series, given after each of the 8 lectures, gave the floor to industrial practitioners using these methods in demanding simulation environments. It highlighted both the achievements of complexity reduction in real applications and the remaining obstacles that must be overcome to meet current industrial needs, in particular in terms of reliability, certification, robustness, and compatibility with modern computing architectures and data-acquisition technologies.

The aim of this International Symposium is now to bring together leading academic researchers in order to present the most recent advances in the field and to identify promising research directions. The talks will be delivered by experts from the academic community, with the intention of providing a state-of-the-art view of current mathematical, numerical, and computational developments. Special attention will be given to methods that combine efficiency with mathematically controlled accuracy, especially through reliable error estimation and certification. This issue is particularly important today, as many approaches inspired by artificial intelligence and machine learning produce impressive numerical results but still lack a complete numerical analysis framework, making it difficult to quantify errors or to design systematic strategies for improving accuracy.

Beyond the lectures themselves, the symposium is intended as a forum for discussion among academic researchers, industrial participants, and students in training. These exchanges should make it possible to challenge and refine the arguments put forward, to clarify the actual contributions of current methods, and to bring to light both the needs arising from applications and the limitations of existing approaches. By gathering specialists in approximation theory, numerical analysis, reduced-order modeling, scientific computing, machine learning, and industrial simulation, the symposium seeks to assess the present state of complexity reduction methods while opening perspectives on emerging methodologies and open problems, including digital twins, real-time optimization, uncertainty quantification, and the simulation of complex physical systems.

For this symposium the support of Collège de France, Inria, Sorbonne Université, Laboratoire Jacques-Louis Lions[1] is greatly acknowledged.

[1] Part of the financial support has been provided by European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant No. 810367), project EMC2

Program

Monday June, 22

Morning	Afternoon
8h50-9h Welcome address by Yvon Maday	
9h-9h45 Virginie Galland Ehrlacher	**14h-14h45** Sebastian Ares de Parga Regalado
9h45-10h30 Benjamin Stamm	**14h45-15h30** Jörg Fehr
10h30-11h *coffee break*	**15h30-16h** *coffee break*
11h-11h45 Angelo Iollo	**16h-16h45** Beatriz Moya
11h45-12h30 Olga Mula	**16h45-17h30** Mario Ohlberger
12h30-14h *lunch break*	**17h30-18h30** *open discussions*

Tuesday June, 23

Morning	Afternoon
9h-9h45 Albert Cohen	**14h-14h45** Bernard Haasdonk
9h45-10h30 Andrea Manzoni	**14h45-15h30** Evie Nielen
10h30-11h *coffee break*	**15h30-16h** *coffee break*
11h-11h45 Benjamin Peherstorfer	**16h-16h45** Helin Gong
11h45-12h30 Clémentine Prieur	**16h45-17h30** Ludovic Chamoin
12h30-14h *lunch break*	**17h30-18h30** *open discussions*

Wednesday June, 24

Morning	Afternoon
9h-9h45 Anthony Nouy	**14h-14h45** David Ryckelynck
9h45-10h30 Geneviève Dusson	**14h45-15h30** Tommaso Taddei
10h30-11h *coffee break*	**15h30-16h15** Karen E. Willcox
11h-11h45 Kathrin Smetana	**16h15-18h00** *coffee break and final discussions*
11h45-12h30 Elise Grosjean	
12h30-14h *lunch break*	

Titles and Abstracts

Sebastian Ares de Parga Regalado

Ludovic Chamoin

Albert Cohen

Geneviève Dusson

Jörg Fehr

Virginie Galland Ehrlacher

Helin Gong

Elise Grosjean

Bernard Haasdonk

Angelo Iollo

Andrea Manzoni

Beatriz Moya

Olga Mula

Evie Nielen

Anthony Nouy

Mario Ohlberger

Benjamin Peherstorfer

Clémentine Prieur

David Ryckelynck

Kathrin Smetana

Benjamin Stamm

Tommaso Taddei

Karen E. Willcox

Sebastian Ares de Parga Regalado

Centre Internacional de Mètodes Numèrics en Enginyeria (CIMNE), Barcelona, Spain.

ROBUST NONLINEAR PROJECTION-BASED REDUCED-ORDER MODELS: COMPARATIVE ASSESSMENT OF CLOSURE AND MANIFOLD STRATEGIES

Abstract

Recent advances in nonlinear model reduction indicate that overcoming linear Kolmogorov limitations requires principled combinations of projection-based approximation and data-driven modeling [3]. In intrusive PROM settings, PROM‑ANN introduced latent-space closure reconstruction of truncated modal coordinates [1], while PROM‑RBF and PROM‑GPR generalized this mechanism through alternative regression operators for the same closure channel [2]. In parallel, projection-compatible nonlinear latent-manifold formulations based on POD-autoencoders (POD-AE), in the spirit of POD-DL-ROM [4], provide an additional pathway to nonlinear approximation while preserving Galerkin/LSPG online dynamics.

The objective of this talk is to establish a coherent comparative framework for robust nonlinear PROM construction across these two families: closure-based reduced representations and manifold-based reduced representations. The analysis addresses central practical questions, including the selection of closure variables, sensitivity to high-mode reconstruction errors, interaction with intrusive residual minimization, and compatibility with hyper-reduction. To this end, we investigate metric-aware basis and closure variants together with computationally scalable intrusive formulations using ECSW hyper-reduction [5].

Numerical investigations are performed on a parametric two-component Burgers benchmark under both in-sample and held-out-parameter evaluations. The comparison is organized around three performance axes−approximation accuracy, computational cost, and achieved speed-up−with the goal of extracting practical guidance on how each strategy can be strengthened and in which operational settings it is most appropriate.

Work with Yvon Maday.

References

[1] J. L. Barnett, C. Farhat, and Y. Maday, J. Comput. Phys., 492:112420, 2023.

[2] S. Ares de Parga, R. Tezaur, C. G. Hernández, and C. Farhat, Comput. Methods Appl. Mech. Eng., 2026.

[3] J. Aghili, H. Ballout, Y. Maday, and C. Prud'Homme, arXiv preprint, arXiv:2601.13712, 2026.

[4] S. Fresca and A. Manzoni, Comput. Methods Appl. Mech. Eng., 388:114181, 2022.

[5] S. Grimberg, C. Farhat, R. Tezaur, and C. Bou-Mosleh, Int. J. Numer. Meth. Eng., 122:1846‑1874, 2021.

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant No. 810367), project EMC2

Ludovic Chamoin

École Normale Supérieure Paris-Saclay (laboratoire LMPS), Saclay, France.

INTEGRATED STRUCTURAL HEALTH MONITORING WITH REAL-TIME DATA ASSIMILATION AND HYBRID TWINS

Abstract

The design of smart autonomous mechanical structures able to perform online control of their integrity, and take anticipated actions during service before downtime or failure occur, has become an active research area. It is a critical need in various industrial sectors (transport, energy, etc.) for more reliability but also more performance and durability of equipment (aircrafts, wind turbines, bridges, etc.). Implementing such an advanced technology would permit optimized maintenance and capability to operate in degraded mode, managing the decrease of loading capabilities by adapting the operating plan.

However, the real-time monitoring of damage in engineering systems, by dynamically coupling predictive simulation tools (in terms of digital twins) and sensor observations, is made very difficult in practice due to several issues. In particular, the complex nonlinear multiscale phenomena which are involved may be associated with computationally intensive simulations (hardly compatible with real-time), which requires reduced order modeling and effective strategies for data assimilation and control. In addition, the problem is plagued with model bias, uncertain environment, and measurement noise, which need to be taken into account for accurate diagnosis and prognosis, and safe decision-making. In this context, an appealing trend is to refer to hybrid twins, in which an a priori physics-guided model is updated and enriched on-the-fly with data-based information, thus making benefit of all knowledge available.

In the talk, I will present some effective numerical methods and recent developments for the construction and use of such hybrid twins, accommodating real-time, accuracy and robustness issues. I will focus on a novel sequential data assimilation framework (based on a modified dual Kalman filter) that continuously integrates sensor data into the numerical model, for damage detection, prediction, and control. I will detail the overall cross-disciplinary methodology which has been chosen, embracing experimental mechanics, data science (including deep learning), advanced modeling and simulation strategies (e.g., use of ROM and adaptive approaches), or computer science. The performance of the proposed strategy will be illustrated on some selected engineering applications e.g, the online control of shaking table tests performed on large-scale damageable concrete buildings, or the design of smart structures equipped with embedded high-resolution distributed optic fiber sensors.

This project has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement No. 101002857)

Albert Cohen

Laboratoire Jacques-Louis Lions, Sorbonne Université, Paris, France.

Optimal linear and non-linear dimensionality reduction

Abstract

Understanding how to optimally approximate general compact sets by finite dimensional spaces is of central interest for designing efficient numerical methods in forward simulation or inverse problems. The concept of n-width, introduced in 1936 by Kolmogorov, is well tailored to linear approximation methods. The interest for n-width has recently been revived by the approximation of parametrized/stochastic PDEs, and the development of reduced basis methods. We briefly survey some now classical results.

We then focus on analogous concepts for nonlinear approximation which are still the object of current research, motivated in particular by the development of neural networks, and possible applications to hyperbolic parametrized PDEs for which linear methods are not effective. We discuss a general framework that allows to embrace various concepts of linear and nonlinear widths, and present some recent results and relevant open problems within this framework.

Geneviève Dusson

Université Marie et Louis Pasteur, CNRS, LmB (UMR 6623), Besançon, France.

Metric-based nonlinear model order reduction with applications to quantum chemistry

Abstract

A broad class of problems in science and engineering involves the repeated solution of partial differential equations (PDEs) for different parameter values. Linear reduced order models are a powerful tool to decrease the computational cost of these simulations by approximating the solutions in a low-dimensional space. They have proven highly effective in many settings; however, they often perform poorly for transport-dominated PDEs, where key solution features such as translations cannot be accurately represented in a linear subspace.

To overcome these limitations, several nonlinear reduced order models have recently been proposed, including approaches based on quadratic or polynomial mappings and neural networks. In this talk, I will present an alternative metric-based approach to nonlinear model order reduction. The central idea is to replace linear combinations in low-dimensional spaces with barycenters taken with respect to a suitably chosen metric, computed from a small number of representative solutions. In particular, I will provide constructions based on the Wasserstein distance from optimal transport, which is well adapted to capturing translations. I will also show how the choice of metric can be adapted to incorporate physical constraints, such as sparsity or prescribed marginals. The proposed methodology will be illustrated through numerical examples involving the approximation of electronic densities and pair densities arising in quantum chemistry.

Jörg Fehr

Institute of Engineering and Computational Mechanics, University of Stuttgart, Pfaffenwaldring 9, 70569 Stuttgart, Germany.

From Latent Space Representations to Practical Surrogate Models for Structural Dynamical Systems

Abstract

The simulation and optimization of complex technical systems often require models that are both sufficiently accurate and computationally efficient. In engineering practice, this balance is difficult to achieve: detailed numerical models provide valuable insight, but they are frequently too costly for repeated evaluations, design optimization, uncertainty studies, or real-time applications.

In this contribution, I will discuss how mathematical methods from model order reduction, system identification, and machine learning can be transferred into practical engineering workflows for structural dynamical systems. The focus is not on replacing physics-based models, but on using data-driven latent space representations to construct surrogate models that remain connected to the underlying mechanical problem.

Several strategies are considered, ranging from black-box latent models to structure-aware identification approaches, including port-Hamiltonian formulations. Particular attention is given to practical issues that arise in technical applications: high-dimensional simulation data, limited or noisy training sets, black-box industrial solvers, multi-physics effects, and the need for reliable predictions beyond isolated benchmark examples.

The methods are illustrated using application-oriented examples such as crash simulations, multiphysics disc-brake models, and further structural and fluid-dynamical systems. The aim is to show how recent mathematical developments can support engineers in analyzing, accelerating, and optimizing complex technical systems while maintaining interpretability and physical plausibility.

Virginie Galland Ehrlacher

École nationale des Ponts et Chaussées, IPP & INRIA, Champs sur Marne, France.

Neural network approximations of solutions of Schrödinger equations

Abstract

Solving eigenvalue problems for high-dimensional Schrödinger operators is a central challenge in numerical analysis, with applications ranging from quantum mechanics to the approximation of complex interacting systems. In this talk, we present a variational approach based on infinite-width two-layer neural networks for the computation of the lowest eigenvalue and associated eigenfunctions of Schrödinger operators with smooth interaction potentials and Neumann boundary conditions on the unit cube.

Using a Barron-type representation of neural network functions by probability measures over parameter space, the energy minimization problem is reformulated as a constrained optimization problem over measures. Following ideas from Wasserstein gradient flows, we introduce and analyze a constrained gradient curve in the space of probability measures. We prove existence of such curves and show that, whenever the dynamics converges, the limiting represented function is an eigenfunction of the Schrödinger operator. This provides a theoretical foundation for neural-network-based variational methods for non-convex eigenvalue problems in high dimension.

Helin Gong

Paris Elite Institute of Technology, Shanghai Jiao Tong University, Shanghai, China.

AI-Driven Complexity Reduction and Multi-Physics Digital Twins: From Theory to Engineering Implementation in Nuclear Reactors

Abstract

To meet the rigorous demands of Best-Estimate Plus Uncertainty (BEPU) in modern nuclear engineering, it is essential to characterize safety margins and system dynamics with both high fidelity and high efficiency. Building upon foundational complexity reduction methods—such as the Generalized Empirical Interpolation Method (GEIM) and Reduced Basis methods—this talk presents the recent advancements in applying these mathematical tools to real-world nuclear engineering practices.

By integrating Model Order Reduction (ROM) with Artificial Intelligence (AI) and Data Assimilation, we have developed a data-enabled, physics-informed digital twin framework. This approach effectively resolves high-dimensional multi-physics coupling problems and allows for ultra-real-time state estimation and parameter identification. Furthermore, the presentation will highlight the engineering implementation of these methodologies, demonstrating how theoretical reduced-order models are deployed into industrial software and platform architectures (e.g., AI-Enhanced Digital Twin Engineering Platform) for the online monitoring and predictive simulation of commercial nuclear reactor cores.

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Elise Grosjean

Inria, Unité de Mathématiques Appliquées, ENSTA, Institut Polytechnique de Paris, 91120 Palaiseau, France

A doubly reduced approximation for the solution to PDEs based on a domain truncation and a reduced basis method: Application to Navier-Stokes equations

Abstract

During this talk, I will present the NIRB two-grid method, together with recent extensions applied to the Navier-Stokes equations, aimed at further reducing the computational cost of the algorithm. The NIRB two-grid method, introduced in [1], is based on two stages. First, during an offline phase, a reduced basis is constructed from high-fidelity solutions computed on a fine mesh, involving a large number of degrees of freedom, using a standard discretisation technique. Then, during the online phase, the parametric problem is solved on a coarser mesh, and the resulting solution is projected onto the reduced space, thereby substantially decreasing the computational cost.

We extend this framework by further reducing the complexity of the online stage. As a representative application, we consider a classical benchmark problem in fluid mechanics: the two-dimensional Backward-Facing Step (BFS). In particular, we simplify the online computation by (i) using a coarse uniform mesh, rather than refining it near the re-entrant corner, and (ii) significantly truncating the outflow section of the channel. Both choices would typically be regarded as detrimental to the accuracy of a high-fidelity flow representation. To overcome this difficulty, we construct two reduced bases and introduce a deterministic linear mapping that enables the transfer from one basis to the other. Additional numerical simulations, including three-dimensional and time-dependent configurations, demonstrate the efficiency of the proposed approach.

[1] R. Chakir and Y. Maday, A two-grid finite-element/reduced basis scheme for the approximation of the solution of parametric dependent P.D.E., in Actes du 9e Colloque national en calcul des structures, Giens, 2009.

Bernard Haasdonk

Institute of Applied Analysis and Numerical Simulation — University of Stuttgart, Stuttgart, Germany.

Greedy Kernel-based Surrogates for Approximating Parametric PDEs

Abstract

Multi-query simulation settings require efficient surrogates for the underlying processes. We consider a scale of kernel-based greedy schemes for approximating the solutions of PDEs, exemplified by elliptic boundary value problems [1]. The procedure is based on an optimal recovery / generalized interpolation framework in reproducing kernel Hilbert spaces and was coined PDE-β-greedy procedure, where the parameter $\beta \geq 0$ is used in a greedy selection criterion and steers the degree of function adaptivity. Algebraic convergence rates have been obtained for Sobolev-space kernels and solutions of finite smoothness [1] and have recently been refined [4]. Exponential convergence rates can be proven for the case of an infinitely smooth kernel and solutions [2].

The scheme can be extended to parametric PDEs by the use of position-parameter product kernels [3]. In the surrogate modelling context, the resulting approach can be interpreted as an a priori model reduction approach, as no solution snapshots need to be precomputed. Numerical results show the efficiency of the approximation procedure for problems which occur as challenges for other parametric MOR procedures: non-affine geometry parametrizations, moving sources or high-dimensional domains.

[1] Wenzel, T., Winkle, D., Santin, G. and Haasdonk, B.: Adaptive meshfree approximation for linear elliptic partial differential equations with PDE-greedy kernel methods. BIT Numerical Mathematics, 65:1, 2025. https://doi.org/10.1007/s10543-025-01053-0.

[2] Vogel, M.-P.: Target dependent greedy sampling for Gaussian kernel PDE collocation, B.Sc. Thesis, University of Stuttgart, 2024. https://doi.org/10.18419/opus-15665

[3] Haasdonk, B., Wenzel, T., Santin, G.: Kernel-based Greedy Approximation of Parametric Elliptic Boundary Value Problems. ACOM, 2026. Accepted. Preprint https://arxiv.org/abs/2507.06731, 2025.

[4] Haasdonk, B., Santin, G., Wenzel, T., Winkle, D.: Refined rates of convergence for target-data dependent greedy generalized interpolation with Sobolev kernels, Applied Mathematics Letters, 2026. Accepted. Preprint arXiv:2601.20407

Angelo Iollo

Institut de Mathématiques de Bordeaux, Université de Bordeaux & Equipe-projet commune MONHADE Inria - Onera - Université de Bordeaux, Bordeaux, France.

Collocated Reduced-Order Models for High-Order Schemes: Analysis and Application

Abstract

In this talk, I will present Collocated Model Order Reduction (cMOR), a novel hyper-reduction framework specifically designed for high-order discretizations. The method evaluates the governing operator exclusively on a small subset of collocation points, identified via Non-Negative Least Squares (NNLS) sparse quadrature. By combining a restricted POD basis with carefully constructed prolongation and interpolation operators, cMOR enables sparse yet accurate residual evaluation while preserving key structural properties of the underlying scheme.

I will discuss the rigorous theoretical foundations of the approach, including proofs of injectivity, optimality of the interpolation error, conditional stability, and convergence in the Lax–Richtmyer sense. These results establish cMOR as a stable and convergent scheme whose accuracy is controlled by the POD and NNLS approximation errors.

Numerical applications to a parametric 2D Shallow Water Equations dam-break problem and a high-order structure-preserving wave equation demonstrate significant computational speedups (up to two orders of magnitude) with negligible loss of accuracy, even in extrapolation regimes.

Work with Giuliano Carlino, Alessia del Grosso, Denis Sipp.

Andrea Manzoni

MOX - Department of Mathematics, Politecnico di Milano, Milano, Italy

Reduced Order Modeling and Scientific Machine Learning: Synergies and Opportunities

Abstract

Among several recently proposed data-driven Reduced Order Models (ROMs), deep learning-based ROMs (DL-ROMs) have proved to be a successful strategy to construct non-intrusive, highly accurate surrogates for the real time solution of parametric nonlinear time-dependent PDEs. By relying on (possibly, convolutional) autoencoders, it is indeed possible to generate latent spaces where the candidate solution is then sought, as a function of parameters and time, using an additional neural network.

In this talk I will provide an overview on DL-ROMs, discussing some recent theoretical results that justify their construction, and connecting them to classical reduced basis methods. Then, I will showcase a series of possible extensions of DL-ROMs capable to (i) handle knowledge of physical laws, (ii) deal with varying geometries, (iii) identify the latent dynamics to ensure accurate out-of-training forecasts, and (iv) include uncertainty quantification.

In all these cases, we will show how the construction of a suitably expressive - and possibly explainable - latent space is essential to ensure accuracy and efficiency of reduced order models exploiting deep neural networks, drawing also some conclusions of possible interest to other contexts in scientific machine learning.

Beatriz Moya

ENSAM Paris, France.

ADVANCES IN HYBRID MODELING: TOWARDS THE NUMERICAL TRANSITION OF TERRITORIES

Abstract

In this talk we will discuss recent advances in hybrid twins and their synergies with artificial intelligence informed by biases that range from physics and geometry to resilience. Examples will be provided in the context of risk assessment and crisis management to develop adapted solutions based on these technologies for the enhancement of the responsiveness of cities and territories...

Olga Mula

Chair of Computational PDEs at the University of Vienna, Austria

GRADIENT FLOWS ON NEURAL NETWORK MANIFOLDS

Abstract

This talk addresses numerical methods for gradient flows in Hilbert spaces based on neural network approximations. The central idea is to represent the solution on a neural network manifold and evolve its parameters in time. At first glance, this approach appears general, elegant, and easy to implement, and it has achieved notable empirical success in machine learning and scientific computing for PDEs. A closer look, however, reveals significant challenges. Developing a proper functional framework that ensures existence of solutions and rigorously connects to practical algorithms raises subtle issues. In this talk, I will present a framework to address these challenges, and show why they are not merely technical obstacles, but rather reflect fundamental aspects of neural approximation.

Evie Nielen

University of Technology Eindhoven, Netherlands.

Efficient Greedy Sampling for Model Order Reduction

Abstract

This talk presents the Polytope Division Method (PDM), a greedy algorithm for solving high-dimensional configuration optimization problems -- such as those arising in model reduction and optimal experimental design -- where one seeks an optimal sampling of parameter spaces. Classical approaches like standard greedy sampling rely on fixed training sets and quickly suffer from the curse of dimensionality. PDM replaces global sampling with an adaptive, geometry-driven strategy based on recursive polytope subdivision. At each step, the method evaluates the objective only at samples in dynamically refined regions. This yields a sampling complexity that scales linearly with dimension, avoiding exponential growth. The approach requires no a priori choice of training set size and focuses computational effort where it matters most. Applications to reduced basis methods and empirical interpolation demonstrate strong performance gains. Numerical results show that PDM achieves comparable accuracy to classical methods at significantly lower offline reduced cost.

Anthony Nouy

Centrale Nantes - Nantes Université, Nantes, France.

STABLE NONLINEAR MANIFOLD APPROXIMATION USING COMPOSITIONAL NETWORKS

Abstract

We consider the problem of approximating a subset M of a Hilbert space X by a low-dimensional manifold M_n. A large class of nonlinear methods can be described by a decoder D: $IR^n \rightarrow X$ whose range is the nonlinear manifold M_n, and an encoder E : $E \rightarrow IR^n$ which extracts n pieces of information $E(u)$ from an element u in M.

Here, we introduce a nonlinear method where E is linear and D is a stable decoder which is obtained by a tree-structured composition of polynomial maps, estimated sequentially from samples in M. Rigorous error and stability analyses are provided, as well as an adaptive strategy for constructing a decoder which guarantees an approximation of the set M with controlled mean-squared or worst-case errors, and a controlled stability (Lipschitz continuity) of the encoder and decoder pair.

Also, we discuss on the definition of optimal encoders and provide concrete strategies for their estimation.

Joint work with A. Bensalah, J. Soffo, A. Somacal.

Mario Ohlberger

Universität Münster — EXC 2044 - Mathematics Münster, Germany.

Reduced Order Surrogate Models for PDE-Constrained Optimization and Inverse Problems

Abstract

Classically, model order reduction for parameterized systems is based on a so-called offline phase, where reduced approximation spaces are constructed and the reduced parameterized system is built, followed by an online phase, where the reduced system can be cheaply evaluated in a multi-query context.

In this contribution, instead, we follow an active learning or enrichment approach where a multi-fidelity hierarchy of reduced order models is constructed on-the-fly while exploring a parameterized system. To this end we focus on learning-based reduction methods in the context of PDE constrained optimization and inverse problems and evaluate their overall efficiency. We discuss learning strategies, such as adaptive enrichment within a trust region optimization framework as well as a combination of reduced order models with machine learning approaches. Concepts of rigorous certification and convergence will be presented, as well as numerical experiments that demonstrate the efficiency of the proposed approaches.

Benjamin Peherstorfer

Courant Institute of Mathematical Sciences, New York University, USA.

Dirac-Frenkel dynamics with momentum

Abstract

Dirac-Frenkel instantaneous residual minimization evolves nonlinear parametrizations of PDE solutions in time, but ill-conditioning can render the parameter dynamics non-unique. We interpret this non-uniqueness as a gauge freedom: nullspace directions that leave the time derivative unchanged can be used to select better-conditioned parameter velocities. Building on Onsager's minimum-dissipation principle, we introduce a history variable (interpretable as momentum) and inject it only along the nullspace directions. The resulting Dirac-Frenkel-Onsager dynamics preserve instantaneous residual minimization, in contrast to standard regularization that can introduce bias, while promoting temporally smooth parameter evolutions. Examples demonstrate that the approach leads to increased robustness in singular and near-singular regimes.

Clémentine Prieur

Université Grenoble Alpes, LJK, Équipe/projet Inria AIRSEA, Grenoble, France.

Diffeomorphism-based feature learning using Poincaré inequalities

Clémentine Prieur (Université Grenoble Alpes), joint work with Romain Verdière (Inria Grenoble) and Olivier Zahm (Inria Grenoble)

Abstract

During this talk, I will present a gradient-enhanced algorithm for high-dimensional function approximation which achieves outperforming accuracy on small data sets.

This algorithm, introduced in [1], proceeds in two steps: first, we reduce the input dimension by learning the relevant input features from gradient evaluations and, second, we regress the function output against the pre-learnt features. Specifically, we learn the feature map by minimizing an error bound obtained using Poincaré inequality applied either in input space or in feature space.

This results in two different strategies which we compare both theoretically and numerically, and which we position in relation to existing methods from the literature. In particular, we prove that if we seek the nonlinear feature map as the first components of a C1-diffeomorphism, then our strategy is theoretically guaranteed. Our strategy to learn the C1-diffeomorphism is based on coupling flows, a particular class of invertible neural networks defined as the composition of block-triangular maps.

Finally I will present several numerical experiments to demonstrate that the algorithm we propose outperforms the state-of-the-art competitors in terms of accuracy with little data sets.

[1] R Verdière, C Prieur, O Zahm : Diffeomorphism-based feature learning using Poincaré inequalities on augmented input space Journal of Machine Learning Research 26 (139), 1-31

David Ryckelynck

Mines Paris - PSL
CEMEF - UMR CNRS 7635 - Mines Paris PSL
1 rue Claude Daunesse BP 207 06904, Sophia Antipolis, France

Self supervised machine learning of ROM-nets for mechanics of materials

Abstract

We propose a general framework for projection-based model order reduction using self-supervised machine learning [1]. For parametric elliptic equations this approach is theoretically based on Céa's Lemma. The proposed methodology, called ROM-net [2], consists in using deep learning techniques to adapt the reduced-order model to a stochastic input tensor whose nonparametrized variabilities strongly influence the quantities of interest for a given physics problem. In particular, we introduce the concept of dictionary-based ROM-nets, where deep neural networks recommend a suitable local reduced-order model from a dictionary. The dictionary of local reduced-order models is constructed from a clustering of vector subspaces in a Grassmann manifold.

It enables the identification of the local low-dimensional subspace in which the solutions evolve for different input tensors. This methodology is applied to an anisothermal elastoplastic problem in structural mechanics coupled to a stochastic thermal field. When using deep neural networks, the selection of the best reduced-order model for a given thermal loading is 60 times faster than when following the clustering procedure used in the training phase. The implementation of local hyper-reduction schemes using a dictionary-based ROM-net is straightforward. The extension to variational inequalities will be addressed at the end of the lecture.

Références

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[2] Model order reduction assisted by deep neural networks (ROM-net) T Daniel, F Casenave, N Akkari, D Ryckelynck Advanced Modeling and Simulation in Engineering Sciences 7 (1), 1-27, (2020)

Kathrin Smetana

Stevens Institute of Technology, Department of Mathematical Sciences, 1 Castle Point Terrace, Hoboken, NJ 07030, USA.

Certified Randomized Model Order Reduction Methods for High-Dimensional Approximation

Abstract

In this talk, we present randomized methods that provide high-probability guarantees for the accuracy of reduced order approximations of parametric partial differential equations (PDEs) with high-dimensional parameter sets. The underlying philosophy is to combine classical reduced basis and greedy approximation ideas with concentration phenomena and data-dependent sampling to obtain certified approximations in high dimensions.

We first present non-asymptotic error bounds for the Proper Orthogonal Decomposition (POD) under the sole assumption that the parameter-to-solution map is uniformly bounded for almost all parameter values. In contrast to existing results, the leading term in our bounds is governed by the sum of the neglected eigenvalues and scales inversely with the number of samples, thereby allowing one to exploit rapid eigenvalue decay. The resulting estimates are independent of the dimension of the parameter space. Consequently, even a modest number of samples can be sufficient for the empirical POD approximation to perform comparably to the ideal POD constructed from the full parameter distribution, including in infinite-dimensional parameter settings.

While the POD result holds for arbitrary sampling strategies, the quality of randomized model reduction can be significantly improved by exploiting problem-dependent sampling. We therefore turn to provably near-optimal sampling strategies. Sampling from Christoffel-type measures provides a near-optimal strategy for identifying large function values from finitely many samples. Building on this idea, we design a randomized greedy algorithm that guarantees, with high probability, the desired approximation accuracy for almost every parameter value in the parameter set, thereby eliminating the need for costly online error assessment. We conclude by showing that, with high probability, the resulting randomized greedy algorithm achieves quasi-optimal convergence rates.

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THE REDUCED BASIS METHOD FOR GROUND-STATE EIGENVALUE COMPUTATIONS

Abstract

In this talk, we present a reduced basis method for the computation of the lowest eigenvalue(s) of a parametrized family of eigenvalue problems motivated by, but not restricted to, many-body quantum problems. Within the reduced basis methods approach, an effective low-dimensional subspace of the high-dimensional Hilbert space is constructed in order to investigate, e.g., the ground-state phase diagram. The basis of this subspace is built from solutions of snapshots, i.e., ground states corresponding to particular and well-chosen parameter values. We highlight the difficulties due to the nature of eigenvalue problems such as degeneracy of eigenstates and vanishing gaps in error certification, and provide some computational strategies with guarantees as potential remedies. We will discuss also recent progress in local error estimation of the Taylor Reduced Basis Method for eigenvalue problems, i.e. if perturbative modes with respect to the parameter are included in the basis as well. Numerical experiments will accompany in both cases the theoretical results.

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Registration in bounded domains for model reduction of parametric conservation laws

Abstract

In this talk, I review recent efforts on the development of registration methods for parametric model order reduction (MOR), with emphasis on advection-dominated flows. In computer vision and pattern recognition, registration refers to the process of finding a parametric transformation that aligns two datasets; in model order reduction, registration methods seek a parametric bijection that tracks coherent structures (e.g., shocks, shear layers) of the solution field. The ultimate goal is to enhance performance of traditional linear compression methods (e.g., POD) and mesh adaptation techniques for the mapped solution field.

We discuss the application of registration techniques to model reduction. First, we illustrate the combination of registration with projection-based reduced-order models and parametric mesh adaptation. Second, we discuss the application of registration to nonlinear interpolation. We present numerical results for two- and three-dimensional parametric compressible flows, to show the potential of the method.

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Multifidelity Proper Orthogonal Decomposition

Abstract

The proper orthogonal decomposition (POD) is widely used to compute a low-dimensional basis that underpins a subsequent dimension reduction or reduced-order modeling step. POD is data-driven in the sense that it requires a training data set of high-fidelity solutions, typically referred to as snapshots. For many complex scientific applications, the computational cost of generating these snapshots is prohibitive, especially when their generation requires sampling over a high-dimensional parameter space. This talk presents a multifidelity POD (mfPOD) formulation that leverages cheaper, lower-fidelity snapshots to reduce the computational cost of computing the POD basis. MFPOD then weights high- and low-fidelity snapshot data via a control-variate formulation to guarantee an unbiased estimate of the expected high-fidelity least-squares projection error. For restrictive computational budgets, the MFPOD cost function has (under some assumptions) lower variance than the POD cost function, which makes the MFPOD subspace more robust against variations in the training data and thus less prone to overfitting. Numerical results show that mfPOD achieves an order of magnitude in computational speedup, translating into useful gains in large-scale problems. Joint work with Nicole Aretz.