

Yvon MADAY's Symposium
June 22-24, 2026, Collège de France

INTEGRATED STRUCTURAL HEALTH MONITORING WITH REAL-TIME DATA ASSIMILATION AND HYBRID TWINS

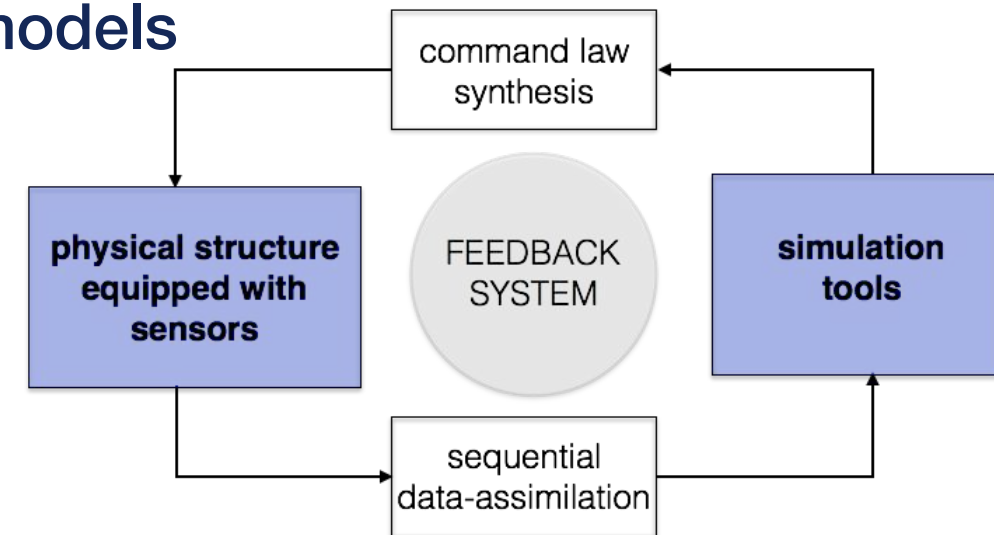
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■ Dynamic Data Driven Application Systems (DDDAS)

- continuous integration of sensor data into numerical models
- customized digital twin to monitor evolving systems
enhanced diagnosis, prognosis & control (anticipate, decide)
- exploitation at best of all knowledge (models & data)
- key enabler of next-generations technologies
- economical & societal impact
increased performance & reliability



integrated simulation-based SHM

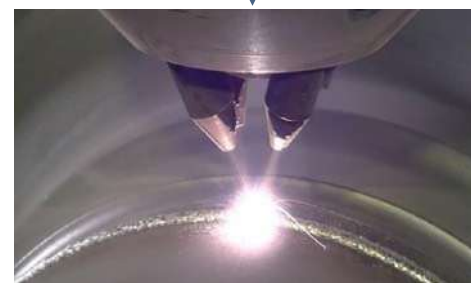


assisted surgery



building performance

**Dynamic Data-Driven
Application Systems
(DDDAS)**



manufacturing



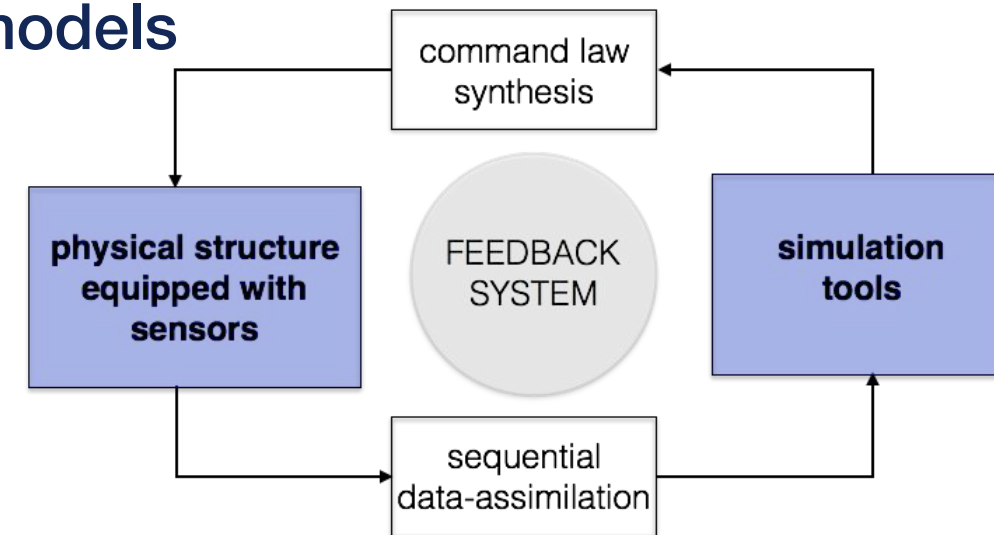
renewable energy



transportation

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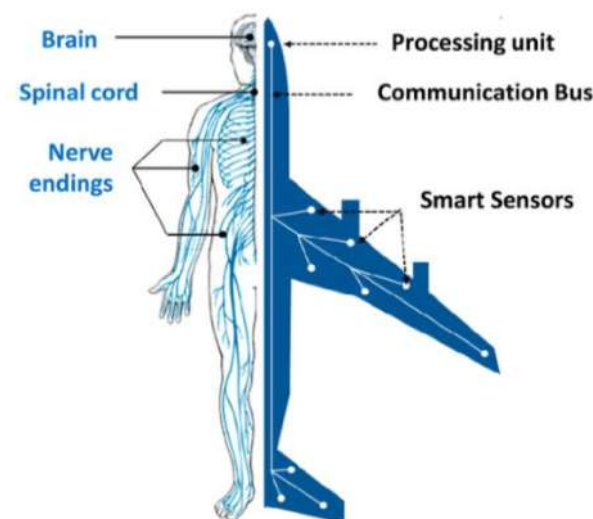


integrated simulation-based SHM

➡ SMART ENGINEERING STRUCTURES

- online monitoring of the integrity (damage detection/control)
- anticipated action during service

➡ *enhanced durability and performance*
operation in degraded mode
optimized maintenance



renewable energy

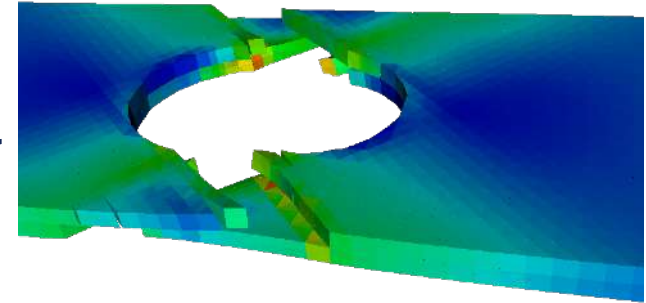


transportation

Challenges & Objectives

■ Challenges for real-life engineering applications

- complex systems (nonlinear, multi-scale/physics, large...)
- limited amount of expensive/heterogeneous/indirect/noisy data
- uncertain environment, biased models of the system behavior



■ Objectives

- numerical methods dedicated to on-the-fly sequential data assimilation & control
- performance in terms of robustness & accuracy (safe decision-making)
- compatibility with advanced sensing & portability (edge computing)

■ Strategy

- use of simplified *a priori* physics-based models with ROM
 - effective correction of model ignorance (data-based enrichment) → **hybrid twins**
 - bias-aware data assimilation & control techniques
- [Chinesta *et al.* 2020]

good prior knowledge + limited amount of data

→ *(multidisciplinary) strategies designed from the mCRE concept*

- **Illustrated introduction to hybrid twins**
- **mCRE framework for data assimilation**
- **Sequential strategy coupling mCRE with Kalman filtering**
Application to the online control of shaking table tests on large-scale damageable concrete buildings
- **On-the-fly adaptive modeling for enhanced computational efficiency**
Application to the design of smart structures equipped with embedded distributed optic fiber sensors

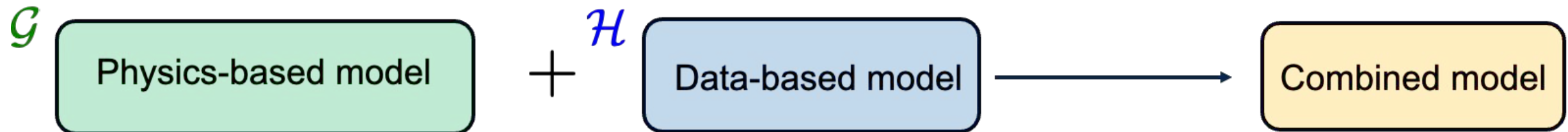
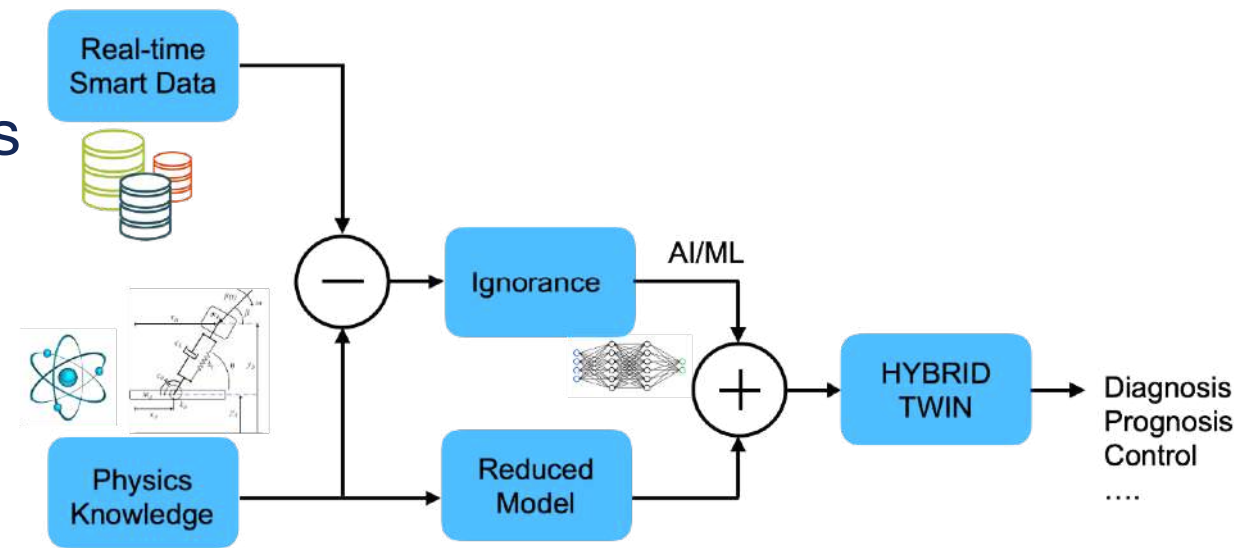
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Basics on Hybrid Twins

uncertain environment/behavior of complex systems

➔ use of *a priori* physics-based (biased) models
e.g. with ROM

➔ data-based correction of model ignorance



$$\tilde{F} = \operatorname{argmin}_{\hat{F} \in \mathcal{G} \oplus \mathcal{H}} \|F - \hat{F}\| \quad \mathcal{G} \oplus \mathcal{H} = \{F_{\mathcal{G}} + F_{\mathcal{H}} : F_{\mathcal{G}} \in \mathcal{G}, F_{\mathcal{H}} \in \mathcal{H}\}$$

- Many advantages:**
- versatility compared to purely data-driven approaches
 - resilience to data overfitting / needs less data
 - performance with conditions outside training bounds (extreme events)
 - interpretability (understanding), explainability (UQ in decision-making),...

BUT the implementation should avoid sub-optimality

- ensure orthogonality/complementarity
- ensure bias-aware data assimilation & control

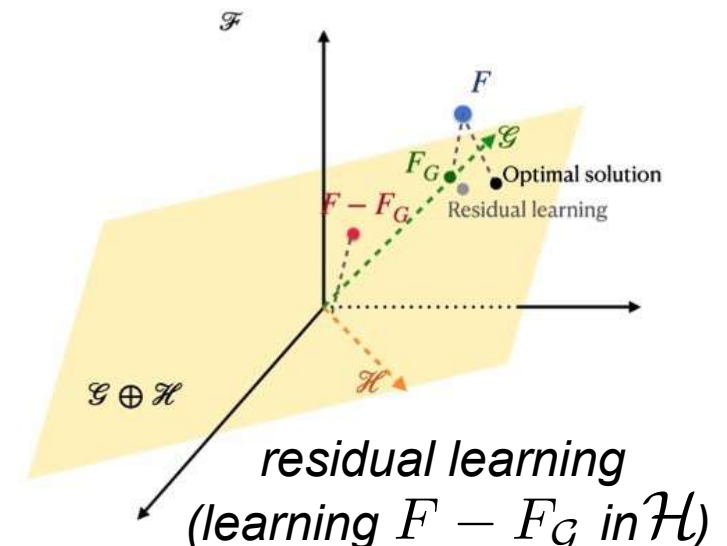
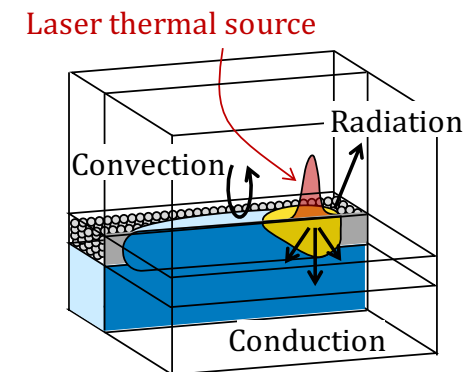


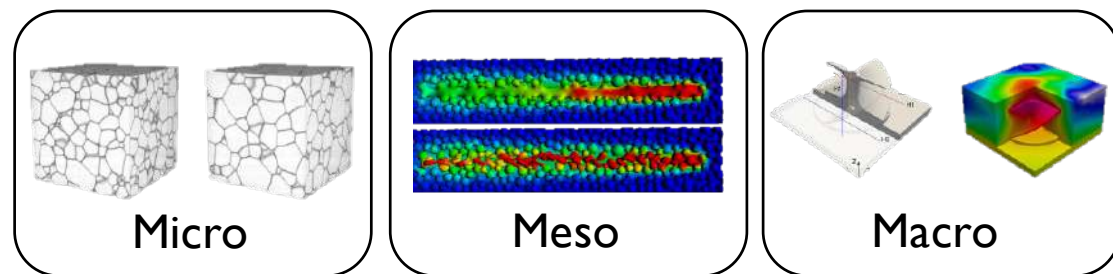
Illustration: Real-Time Monitoring in AM

■ Transient thermal analysis in LPBF processes



GOALS:

- **at melt pool scale:** predict the size of the melt pool depending on the laser trajectory
- **at layer scale:** predict cooling rates and hot points to assess residual stresses



Credits : Cemef, MSSMAT, LMT

complex multiphysics, multiscale phenomena

➡ reduced complexity (simplified model & ROM)
large model bias



thermography data from NIST

Nat. Institute of Standards & Technology (www.nist.gov)

➡ rich (noisy) data to enrich the physics model
selection of relevant data

coarse initial model + rich data ➡ Parametrized Background Data Weak (PBDW)
[Maday et al. 2015]

■ Basic formulation

Non-intrusive / reduced-basis / variational approach

Key idea: seek an approximation of the true state by employing projection-by-data

$$u_{N,M} = z_{N,M} + \eta_{N,M} \quad \text{physics-based estimate: computed from ROM on a surrogate model}$$

data-based correction: informed by experimental observations

Inputs & Outputs (OFFLINE)

- Parametrized best-knowledge model (PDE)

$$\rho_{eq} c \left(\frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u \right) = \lambda_{eq} \Delta T + r$$

- Parametric domain: $\mu = \lambda_{eq} \in D$

→ manifold of solutions $M^{bk} = \{u^{bk,\mu} | \mu \in D\}$



- Space/(time)/parameter decomposition

$$\sum_{n=1}^N \zeta_n(x) \times \lambda_n(t) \times \alpha_n(\mu)$$

LMPS → background space $Z_N = \text{span}\{\zeta_n\}_{n=1}^M$

- Hilbert space (with norm & inner product)

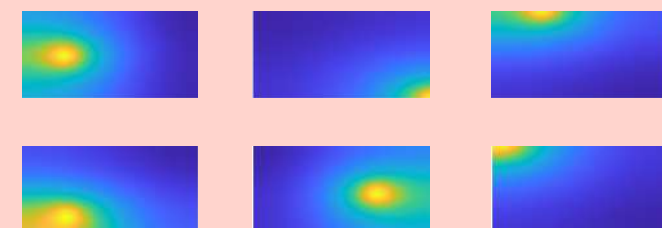
- Positions of sensors: x_m^{obs}

- Observation functionals

$$l_m(v) = C(x_m^{obs}) \int v(x) e^{-\frac{\|x - x_m^{obs}\|^2}{2r_w^2}} dx$$

- Riesz representers: $q_m = R_U l_m$

$$(q_m, v) = l_m(v) \quad \forall v \in U$$



→ update space $U_M = \text{span}\{q_m\}_{m=1}^M$ **6**

Basic formulation

$$u_{N,M} = z_{N,M} + \eta_{N,M}$$

$$\sum_{n=1}^N a_n \zeta_n \in Z_N$$

$$\sum_{m=1}^M b_m q_m \in U_M$$

Implementation (ONLINE)

$$(z_{N,M}, \eta_{N,M}) = \underset{z \in Z_N, \eta \in U_M}{\operatorname{argmin}} \left(\underbrace{\epsilon \|\eta\|^2}_{\text{modeling error}} + \underbrace{\|L(z + \eta) - y^{\text{obs}}\|_2^2}_{\text{data misfit}} \right)$$

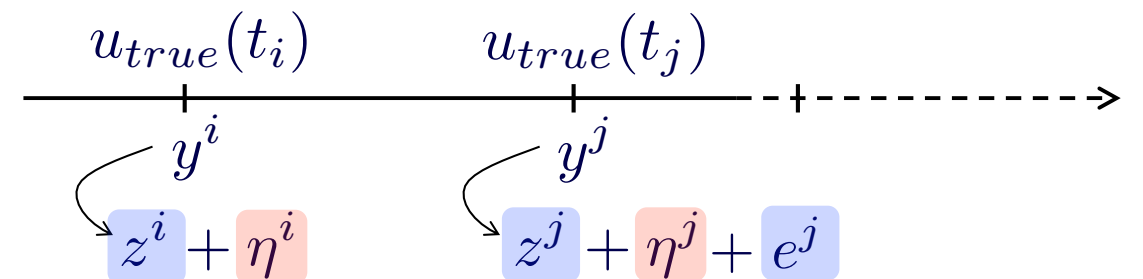
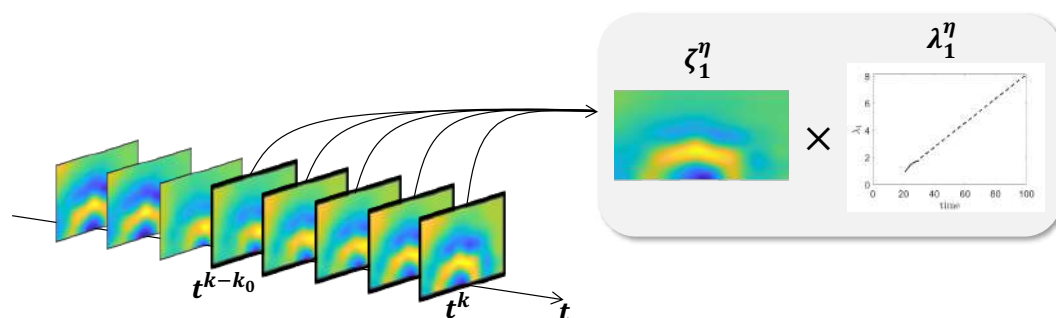
$$\hookrightarrow \begin{bmatrix} \mathbb{A} + \epsilon M \mathbb{I} & \mathbb{B} \\ \mathbb{B}^T & 0 \end{bmatrix} \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} y^{\text{obs}} \\ 0 \end{pmatrix} \quad \begin{aligned} A_{i,j} &= (q_i, q_j) \\ B_{i,j} &= (\zeta_i, q_j) \end{aligned}$$

- full error convergence analysis provided [Gong *et al.* 2019]
- embedded optimal data management (stability-greedy algorithm, cf. GEIM)

Step 1: optimal sensor placement (description of the background space): $\max \beta_{N,M} = \inf_{\eta \in U_M} \sup_{z \in Z_N} \frac{|(z, \eta)|}{\|z\| \cdot \|\eta\|}$

Step 2: farthest first traversal to get a good approximation of η : $x_{m+1}^{\text{obs}} = \underset{x \in X}{\operatorname{argmax}} \min_{i=1:m} \|x - x_i^{\text{obs}}\|$

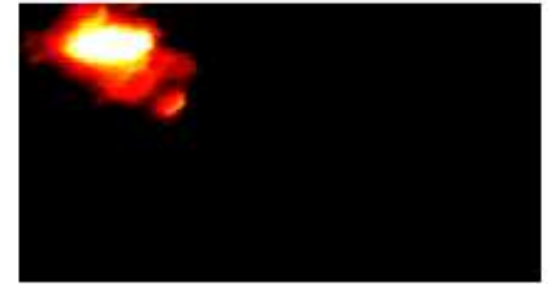
- extension to parametrized time-dependent problems [Haïk *et al.* 2023]



Monitoring at the Layer Scale

■ Experimental data

- Thermography measurements from NIST [Molnar *et al.* 2021]
- High-frequency global camera ($f = 2000Hz$)



■ Surrogate (background) model

- 2D analytical thermal model from the FLASH method [Ettaieb *et al.* 2021]

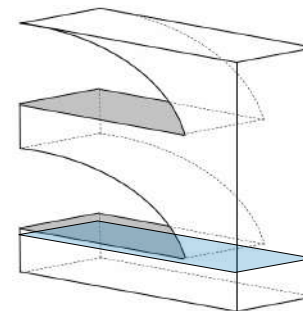
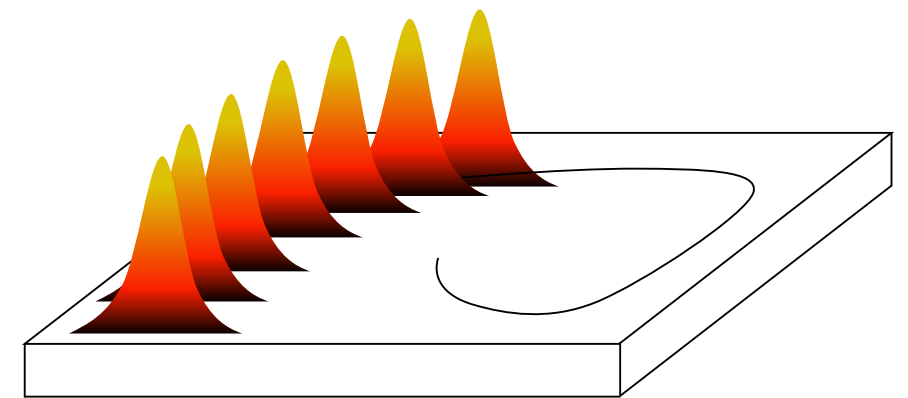
Effect of a single flash

$$T_f(t, r, \alpha, \epsilon) = \frac{2AQ}{\epsilon\sqrt{\pi^3(t)}} \frac{1}{R^2 + 8\alpha(t)} e^{-\frac{2r^2}{R^2 + 8\alpha(t)}}$$

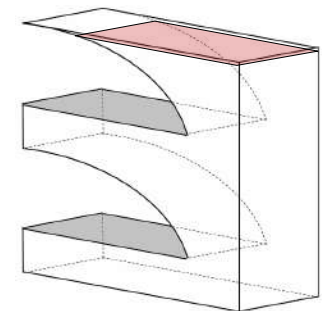
Convolution along the trajectory $s(t)$

$$T(t, x) = \sum_i (\gamma_i \times \Delta t) \cdot T_f(t - i\Delta t, \|x - s_i\|_2, R)$$

- Conductivity λ plays a major role in the t° profile



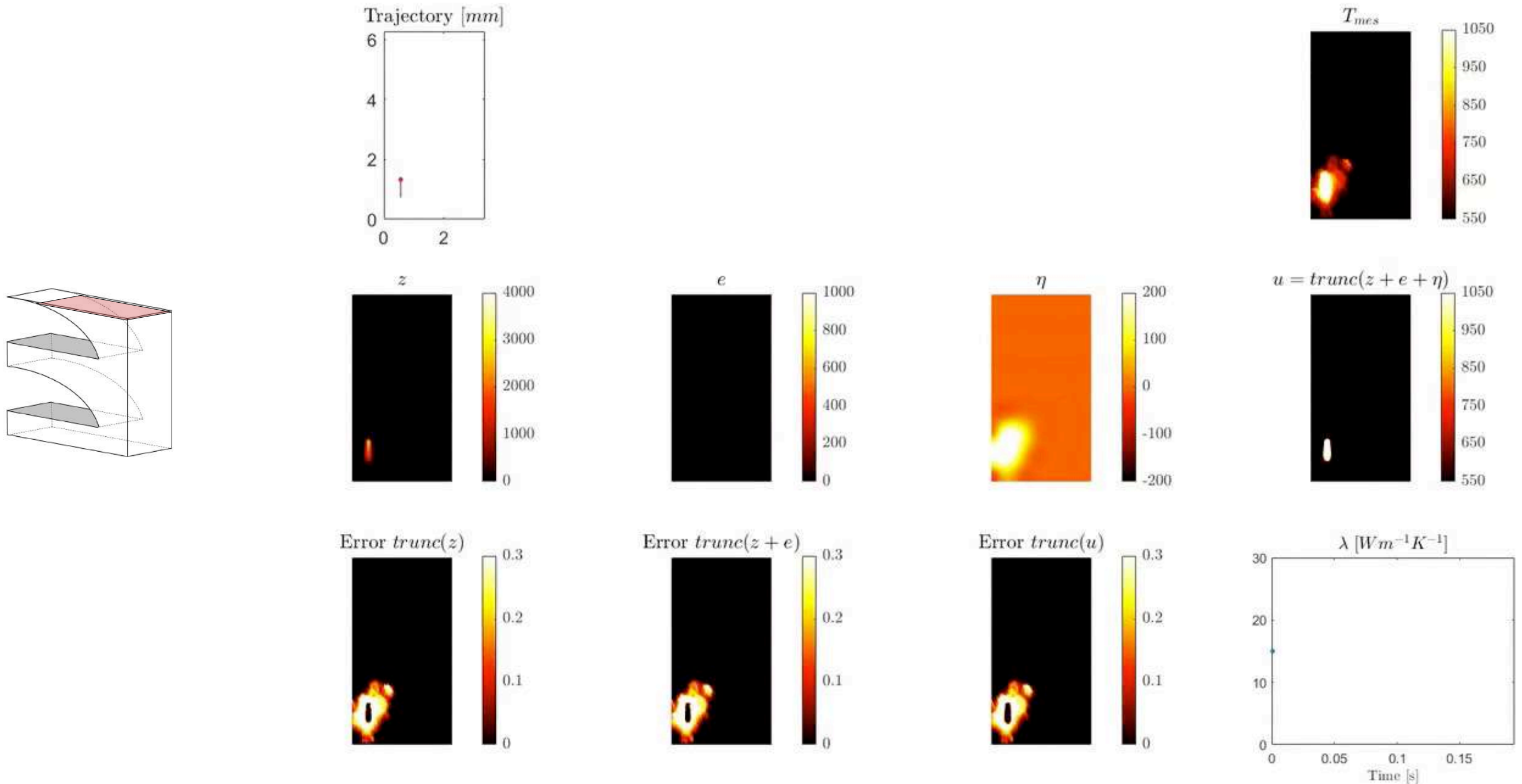
Layer on a massive part
(built on previous molten layer)



Layer on an overhang
(mostly built on powder)

Monitoring at the Layer Scale

■ Results



Monitoring in real-time using parallel computation: $t_{est} \sim 5 - 10ms$

Ongoing work: learn the bias from AI

(Physics-enhanced ML, PBDW+DeepONets [Massala *et al.* 2026])

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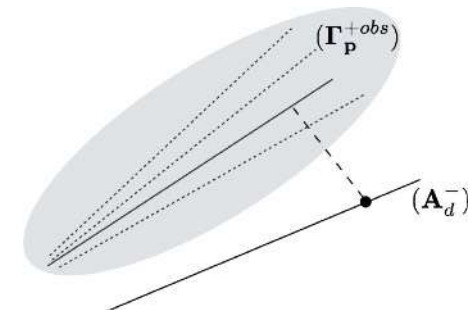
■ Reliability of information (various error sources)

The problem is split in : - reliable info (equilibrium,...) → enforced in the process (admissibility)

- unreliable info (material behavior, BCs, sensor values,...)

Cost function: $\mathcal{E}_{mCRE}(\hat{\mathbf{u}}, \hat{\sigma}; \mathbf{p}) = \underbrace{\mathcal{E}_{CRE}^2(\hat{\mathbf{u}}, \hat{\sigma}; \mathbf{p})}_{\text{modeling error term}} + \frac{\alpha}{2} \underbrace{(\mathbf{d}(\hat{\mathbf{u}}) - \mathbf{d}_{obs})^T \mathbb{G}_{obs}^{-1} (\mathbf{d}(\hat{\mathbf{u}}) - \mathbf{d}_{obs})}_{\text{distance to observations}}$

$$\hookrightarrow \mathbf{p}_{sol} = \underset{\mathbf{p} \in \mathcal{P}}{\operatorname{argmin}} \left[\min_{(\hat{\mathbf{u}}, \hat{\sigma}) \in (\mathbf{A}_d^-)} \mathcal{E}_{mCRE}(\hat{\mathbf{u}}, \hat{\sigma}; \mathbf{p}) \right]$$



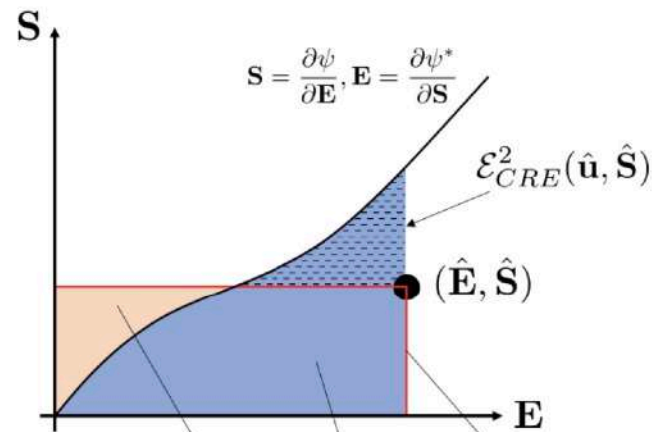
- regularization from physics with various applications
variational primal-dual formulation [Chavent et al. 1999], Kohn-Vogelius-type functional [Kohn & Lowe 1988]
- nice mathematical properties (enhanced convexity, spatial localization of defects...)
- robustness with noisy/corrupted data [Feissel & Allix 2007, Ben Azzouna et al. 2015]
- explicit (distributed) modeling error to drive/learn the model complexity

Link with Bayesian inf.: $\pi(\mathbf{d}_{obs} | \mathbf{p}) = C_1 . e^{-\frac{1}{2} (\mathbf{d}_{obs} - \mathbf{d}(\hat{\mathbf{u}}(\mathbf{p})))^T \mathbb{G}_{obs}^{-1} (\mathbf{d}_{obs} - \mathbf{d}(\hat{\mathbf{u}}(\mathbf{p})))} . e^{-\frac{\mathcal{E}_{CRE}^2(\hat{\mathbf{u}}, \hat{\sigma}; \mathbf{p})}{\alpha}}$ π_{mod}

(Tarantola's framework)

■ CRE measure [Ladevèze & Pelle 2004, Ladevèze & LC 2015]

$$\mathbf{S} = \frac{\partial \psi}{\partial \mathbf{E}} \rightarrow \mathcal{E}_{CRE}^2(\hat{\mathbf{u}}, \hat{\mathbf{S}}) = \int_{\Omega} \left(\psi(\hat{\mathbf{u}}) + \psi^*(\hat{\mathbf{S}}) - \hat{\mathbf{S}} : \mathbf{E}(\hat{\mathbf{u}}) \right)$$



$$\psi^*(\hat{\mathbf{S}}) = \sup_{\mathbf{E}} [\hat{\mathbf{S}} : \mathbf{E} - \psi(\mathbf{E})]$$

Legendre-Fenchel dual potential

• Case of quadratic potentials

$$\psi(\mathbf{E}(\hat{\mathbf{u}})) = \frac{1}{2} \mathbf{E}(\hat{\mathbf{u}}) : \mathbf{K} : \mathbf{E}(\hat{\mathbf{u}}) \quad \psi^*(\hat{\mathbf{S}}) = \frac{1}{2} \hat{\mathbf{S}} : \mathbf{K}^{-1} : \hat{\mathbf{S}}$$

$$\mathcal{E}_{CRE}^2(\hat{\mathbf{u}}, \hat{\mathbf{S}}) = \frac{1}{2} \int_{\Omega} (\hat{\mathbf{S}} - \mathbf{K} \mathbf{E}(\hat{\mathbf{u}})) : \mathbf{K}^{-1} : (\hat{\mathbf{S}} - \mathbf{K} \mathbf{E}(\hat{\mathbf{u}}))$$

$$\mathcal{E}_{CRE}^2(\hat{\mathbf{u}}, \hat{\mathbf{S}}) = \int_{\Omega} \left(\psi(\hat{\mathbf{u}}) + \psi^*(\hat{\mathbf{S}}) - \hat{\mathbf{S}} : \mathbf{E}(\hat{\mathbf{u}}) \right)$$

➡ related to sym. Bregman divergence (discrepancy metric)

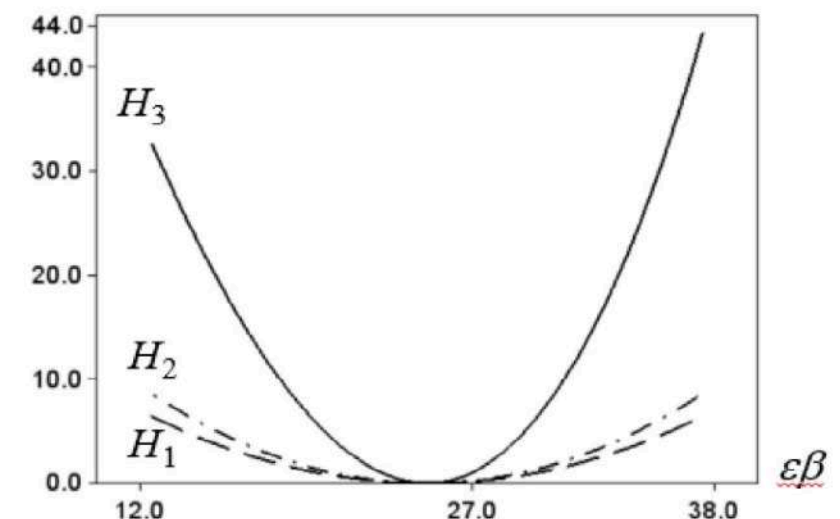
S. Andrieux, Bregman divergences for physically informed discrepancy measures for learning and computation in thermomechanics, *Comptes Rendus Mécanique*, **351**:59-81 (2023)

$$-u'' + \alpha^2 u + \varepsilon \beta u^3 = 0, \quad u(0) = 0, u'(1) = C$$

$$H_1(\varepsilon \beta) = \frac{\alpha^2}{2} \int_0^1 (u(\varepsilon \beta) - u(\varepsilon \beta_0))^2 dx,$$

$$H_2(\varepsilon \beta) = \frac{1}{2} \int_0^1 [(u'(\varepsilon \beta) - u'(\varepsilon \beta_0))^2 + \alpha^2 (u(\varepsilon \beta) - u(\varepsilon \beta_0))^2 + \varepsilon \beta (u(\varepsilon \beta) - u(\varepsilon \beta_0))^4] dx$$

$$H_3(\varepsilon \beta) = \frac{1}{2} \int_0^1 [(u'(\varepsilon \beta) - u'(\varepsilon \beta_0))^2 + \alpha^2 (u(\varepsilon \beta) - u(\varepsilon \beta_0))^2 + \varepsilon \beta (u(\varepsilon \beta) - u(\varepsilon \beta_0))(u^3(\varepsilon \beta) - u^3(\varepsilon \beta_0))] dx.$$



■ Implementation - Elasticity case

• **Step 1 (localization):** define $\mathcal{F}(\mathbf{p}) = \min_{(\hat{\mathbf{u}}, \hat{\sigma}) \in (\mathbf{A}_d^-)} \mathcal{E}_{mCRE}(\hat{\mathbf{u}}, \hat{\sigma}; \mathbf{p})$

→ Lagrangian functional $\mathcal{L}(\hat{\mathbf{u}}, \hat{\sigma}, \boldsymbol{\lambda}; \mathbf{p}) = \mathcal{E}_{mCRE}^2(\hat{\mathbf{u}}, \hat{\sigma}; \mathbf{p}) - \left[\int_{\Omega} \hat{\sigma} : \epsilon(\boldsymbol{\lambda}) - \int_{\Omega} \mathbf{f}_d \cdot \boldsymbol{\lambda} - \int_{\partial_2 \Omega} \mathbf{F}_d \cdot \boldsymbol{\lambda} \right]$

↘ $\mathbf{K}\epsilon(\mathbf{v})$ (duality)

→ $\mathcal{L}(\mathbf{U}, \mathbf{V}, \boldsymbol{\Lambda}; \mathbf{p}) = \frac{1}{2}(\mathbf{U} - \mathbf{V})^T \mathbb{K}(\mathbf{p})(\mathbf{U} - \mathbf{V}) + \frac{\alpha}{2}(\Pi \mathbf{U} - \mathbf{U}_{obs})^T \mathbb{G}_{obs}(\Pi \mathbf{U} - \mathbf{U}_{obs}) - \boldsymbol{\Lambda}^T (\mathbb{K}(\mathbf{p})\mathbf{V} - \mathbf{F})$

(saddle point)

→ $\begin{bmatrix} \alpha \Pi^T \mathbb{G}_{obs} \Pi & -\mathbb{K} \\ \mathbb{K} & \mathbb{K} \end{bmatrix} \begin{pmatrix} \mathbf{U} \\ \boldsymbol{\Lambda} \end{pmatrix} = \begin{pmatrix} \alpha \Pi^T \mathbb{G}_{obs} \mathbf{U}_{obs} \\ \mathbf{F} \end{pmatrix} \Rightarrow (\mathbb{K} + \alpha \Pi^T \mathbb{G}_{obs} \Pi) \mathbf{U} = \mathbf{F} + \alpha \Pi^T \mathbb{G}_{obs} \mathbf{U}_{obs}$

data-based model enrichment (**hybrid twin**)

$\mathbf{V} = \mathbf{U} + \boldsymbol{\Lambda}$

• **Step 2 (correction):** solve $\min_{\mathbf{p} \in \mathcal{P}} \mathcal{F}(\mathbf{p})$ (*hierarchic updating*)

→ gradient-based approach with adjoint state method ($\nabla_{\mathbf{p}} \mathcal{F} = \frac{\partial}{\partial \mathbf{p}} \mathcal{L}(\mathbf{U}, \mathbf{V}, \boldsymbol{\Lambda}; \mathbf{p})$)
(steepest descent, Gauss-Newton, BFGS,...)

• Multi-query context → use of ROM to compute $\hat{\mathbf{s}}(\mathbf{x}, t, \mathbf{p}, \alpha, \mathbf{u}_{obs}, \dots)$ (POD, LATIN-PGD)

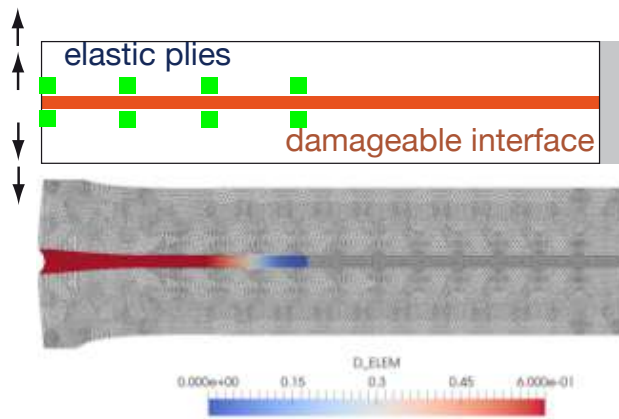
[Bouclier *et al.* 2013, Marchand *et al.* 2016, Bhattacharyya & LC 2026]

mCRE Framework

■ Nonlinear constitutive laws (generalized standard materials [Halphen & Nguyen 1975])

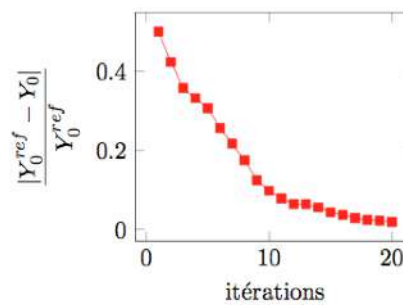
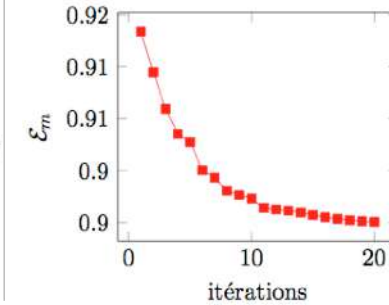
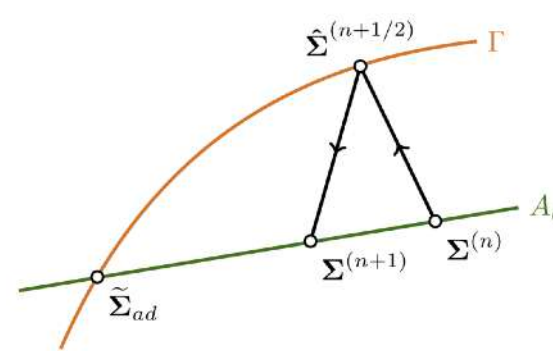
use of 2 dual thermodynamics potentials / CRE measures from Legendre-Fenchel residuals

$$\underset{\text{state equations}}{s = \frac{\partial \psi}{\partial \mathbf{e}_e}} ; \underset{\text{evolution laws}}{s = \frac{\partial \varphi}{\partial \dot{\mathbf{e}}_p}} \quad \boxed{\mathcal{E}_{CRE|t}^2 = \int_{\Omega} \eta_{\psi}(\hat{\mathbf{e}}_e, \hat{\mathbf{s}}) + \int_0^t \int_{\Omega} \eta_{\varphi}(\dot{\mathbf{e}}_p, \hat{\mathbf{s}})} \quad \text{with} \quad \begin{cases} \eta_{\psi}(\hat{\mathbf{e}}_e, \hat{\mathbf{s}}) = \psi(\hat{\mathbf{e}}_e) + \psi^*(\hat{\mathbf{s}}) - \hat{\mathbf{s}} \cdot \hat{\mathbf{e}}_e \\ \eta_{\varphi}(\dot{\mathbf{e}}_p, \hat{\mathbf{s}}) = \varphi(\dot{\mathbf{e}}_p) + \varphi^*(\hat{\mathbf{s}}) - \hat{\mathbf{s}} \cdot \dot{\mathbf{e}}_p \end{cases}$$



$$\dot{d} = \frac{k}{a} \left(1 - \exp \left[-a \left\langle \frac{Y - Y_0}{Y_c - Y_0} - d^2 \right\rangle_+ \right] \right)$$

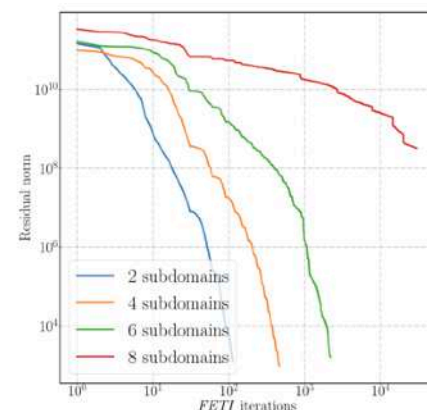
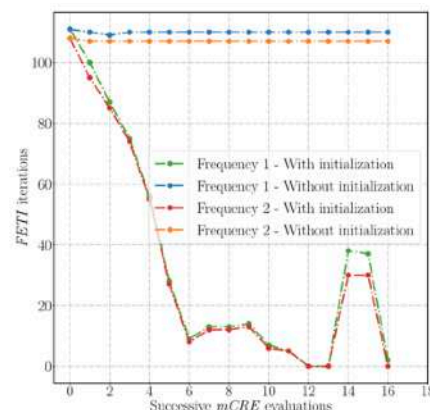
[Marchand et al. 19]



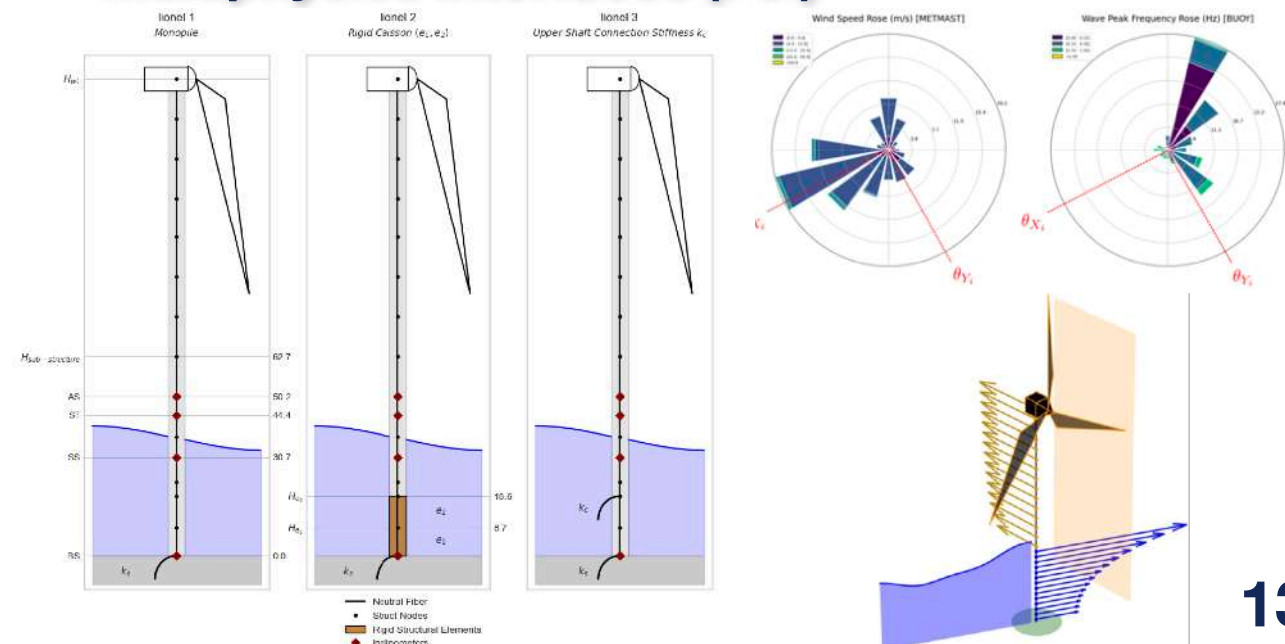
■ Some recent implementations



HPC with domain decomposition [Samir et al. 2023]

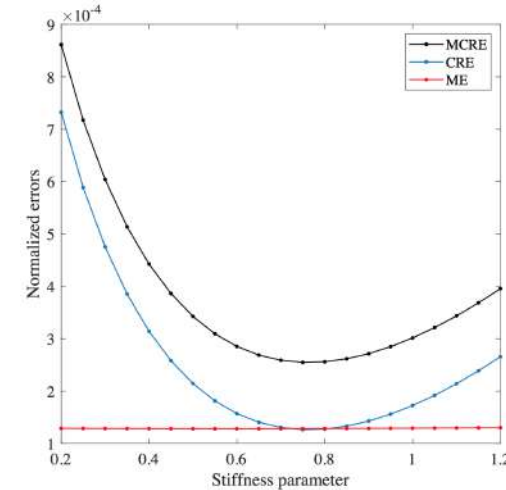
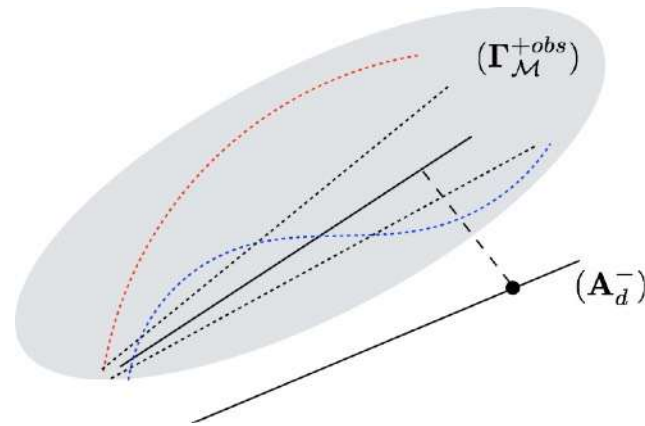


Multiphysics interfaces (FSI) [Roussel et al. 2023]



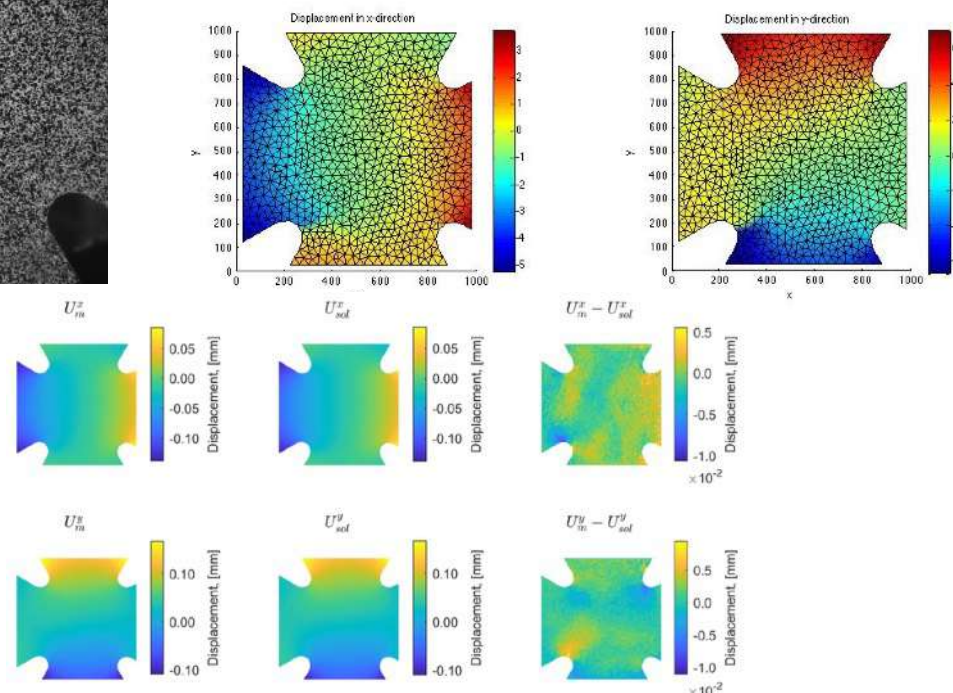
■ Data-based model selection [Nguyen *et al.* 2022, Nguyen & LC 2024]

GOAL: select a reference model which is **consistent with the richness of observations**



mIDIC (integrated version):

$$\mathcal{E}_{mIDIC}^2(\hat{\mathbf{U}}, \hat{\mathbf{V}}, \mathbf{p}) = \underbrace{\frac{1}{2}(\hat{\mathbf{U}} - \hat{\mathbf{V}})^T \mathbb{K}(\mathbf{p})(\hat{\mathbf{U}} - \hat{\mathbf{V}})}_{\text{CRE term (modeling error)}} + \underbrace{\frac{\alpha}{2} \cdot \frac{1}{2\gamma_f^2 N_{pix}} \sum_{\mathbf{x} \in ROI} \left(f(\mathbf{x}) - g(\mathbf{x} + \mathbf{N}(\mathbf{x})\hat{\mathbf{U}}) \right)^2}_{\text{correlation residual (gray level conservation)}}$$

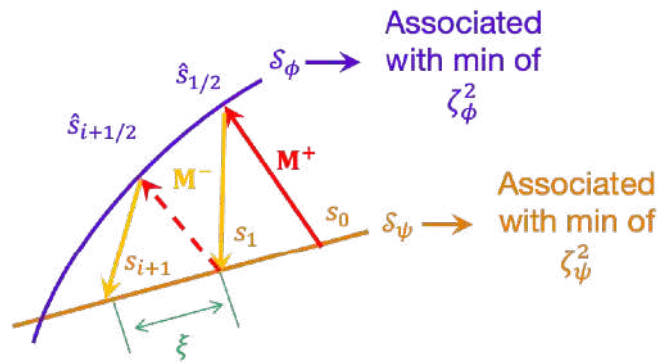


MODEL	$\overline{E}_{CRE}^2$
Initial isotropic elasticity	17.56
Orthotropic elasticity	3.27
Elasto-plasticity (Prandtl-Reuss)	2.29
Adapted mesh	1.38

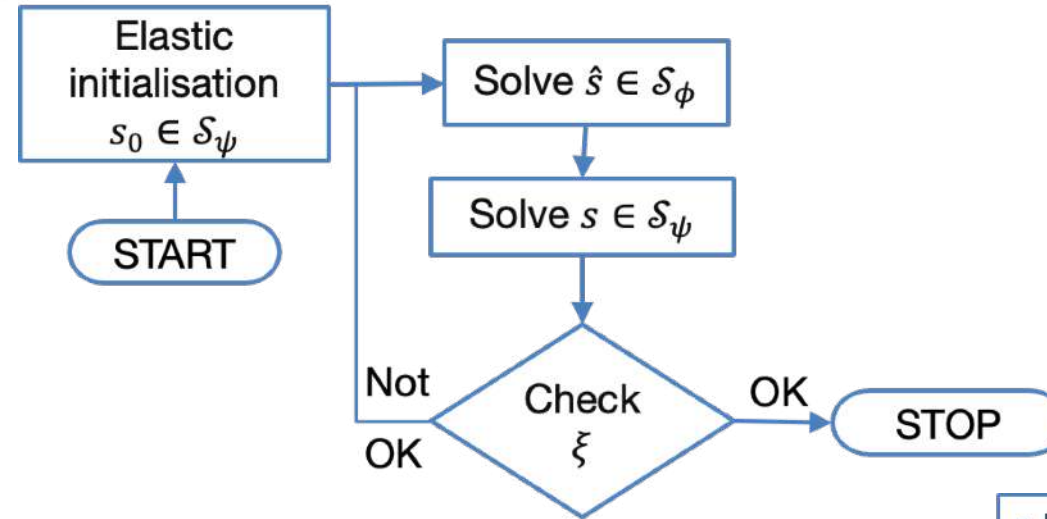
Reduced Order Modeling

[Bhattacharyya & LC 2026]

LATIN-PGD type approach



$$s = \{\epsilon_u, \epsilon_v\} \quad \hat{s} = \{\sigma_u, \sigma_v, \epsilon_u^p, \epsilon_v^p, X_u, X_v, Y_u, Y_v\}$$

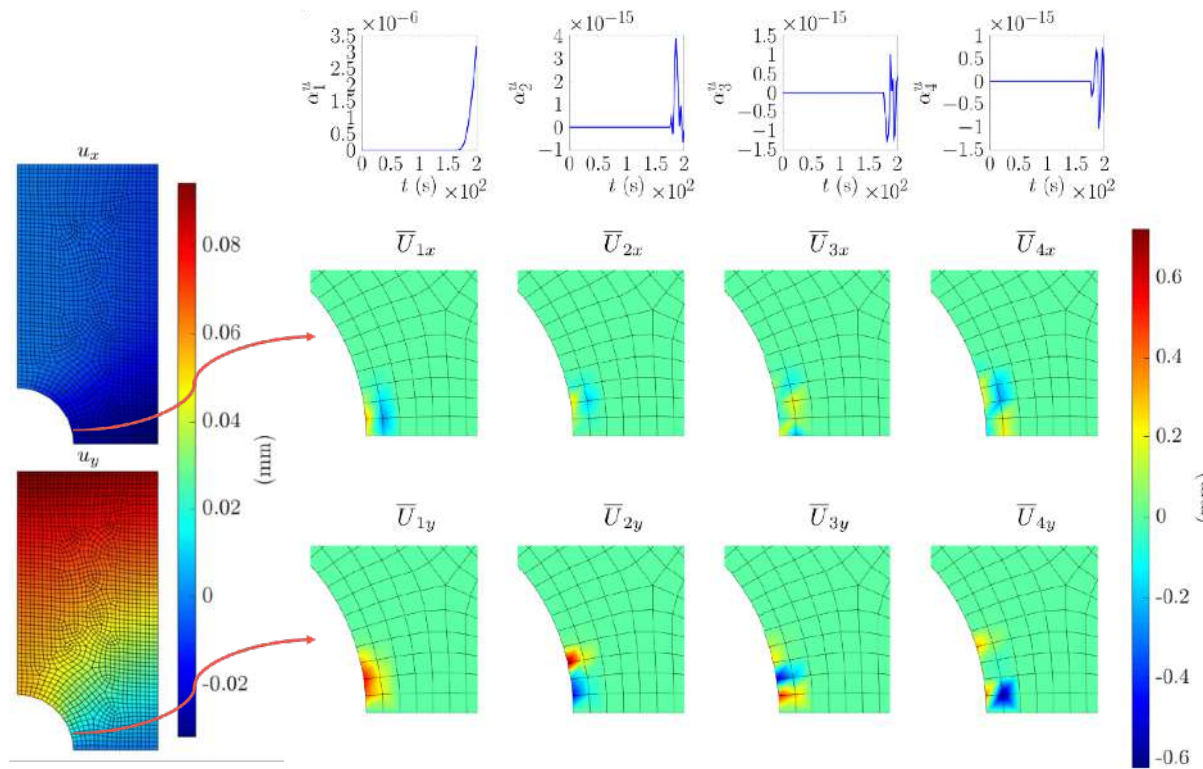
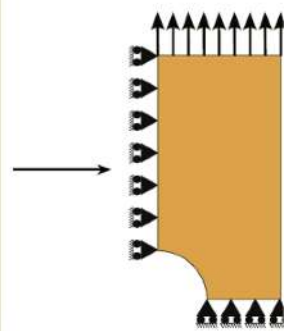
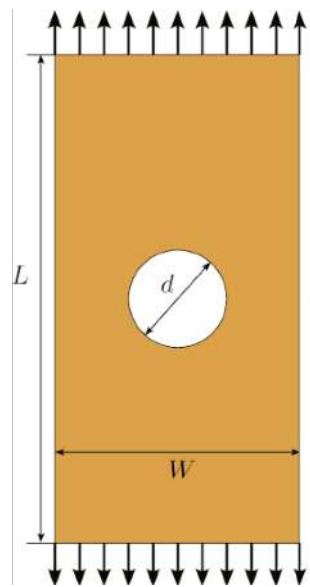


Corrective terms

- $\Delta \underline{v}_{i+1} = \underline{v}_{i+1} - \underline{v}_i$
- $\Delta \underline{u}_{i+1} = \underline{u}_{i+1} - \underline{u}_i$
- $\Delta \underline{\lambda}_{i+1} = \underline{\lambda}_{i+1} - \underline{\lambda}_i$
- $\Delta \epsilon_{u,i+1} = \epsilon_{u,i+1} - \epsilon_{u,i}$
- $\Delta \epsilon_{v,i+1} = \epsilon_{v,i+1} - \epsilon_{v,i}$
- $\Delta \epsilon_{\lambda,i+1} = \epsilon_{\lambda,i+1} - \epsilon_{\lambda,i}$

Low-dimensional approximation of full high-dimensional systems

$$\underline{u}_i(\underline{x}, t) = \underline{u}_0 + \sum_{j=1}^{m_u} \alpha_j^u(t) \bar{U}_j(\underline{x}), \quad \underline{\lambda}_i(\underline{x}, t) = \underline{\lambda}_0 + \sum_{j=1}^{m_\lambda} \alpha_j^\lambda(t) \bar{\Lambda}_j(\underline{x})$$



State laws

- $\sigma = \mathbf{C}(E, \nu) : \epsilon^e$
- $R = R_\infty p$

Evolution laws

- $\dot{\epsilon}^p = \left\langle \frac{f^p}{K} \right\rangle^n \left[\frac{3}{2} \frac{\sigma^D}{\sqrt{\frac{3}{2} \sigma^D : \sigma^D}} \right]$
- $\dot{p} = \left\langle \frac{f^p}{K} \right\rangle^n$

Update of ROB

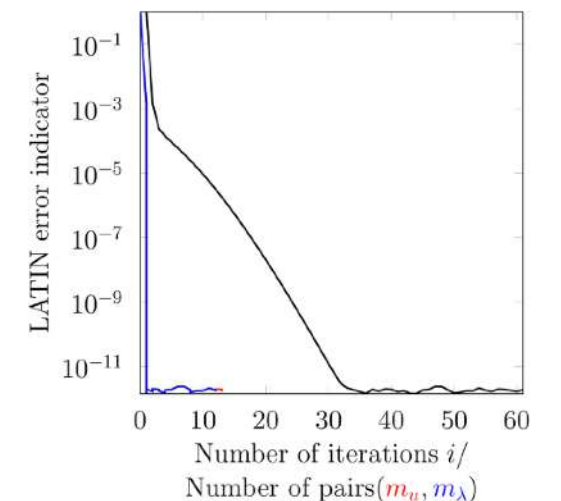
- Variable separation

$$\Delta \underline{u}_{i+1} = \sum_{j=1}^{m_u} \Delta \alpha_j^u(t) \bar{U}_j(\underline{x})$$

$$\Delta \underline{\lambda}_{i+1} = \sum_{j=1}^{m_\lambda} \Delta \alpha_j^\lambda(t) \bar{\Lambda}_j(\underline{x})$$

- Time functions

$$\{\Delta \alpha_j^u\}_{j=1}^{m_u}, \{\Delta \alpha_j^\lambda\}_{j=1}^{m_\lambda} = \arg \min_{\{\Delta \alpha_j^u\}_{j=1}^{m_u}, \{\Delta \alpha_j^\lambda\}_{j=1}^{m_\lambda}} \mathcal{L}$$



Error estimate available
[Chamoin et al. 2017]

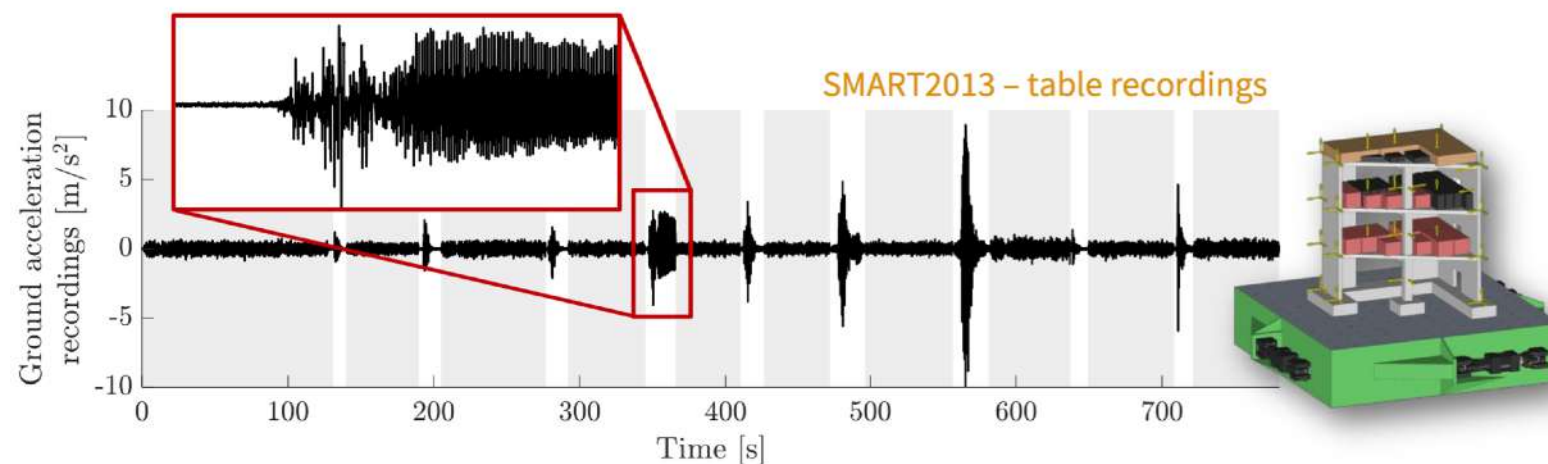
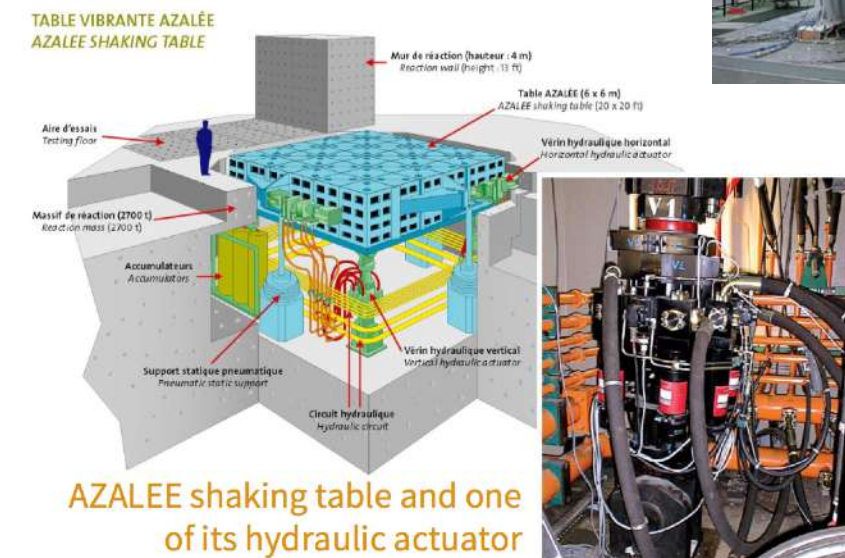
- Illustrated introduction to hybrid twins
- mCRE framework for data assimilation
- **Sequential strategy coupling mCRE with Kalman filtering**
Application to the online control of shaking table tests on large-scale damageable concrete buildings
- **On-the-fly adaptive modeling for enhanced computational efficiency**
Application to the design of smart structures equipped with embedded distributed optic fiber sensors

Application: SHM on Concrete Buildings

■ Automated control of dynamics shaking-table tests



- low-frequency dynamics (earthquake engineering)
- SMART 2013 benchmark (CEA/EDF)
- sequence of gradually damaging tests on a concrete structure
- command on hydraulic actuators
- if inappropriate control, unstable experiment



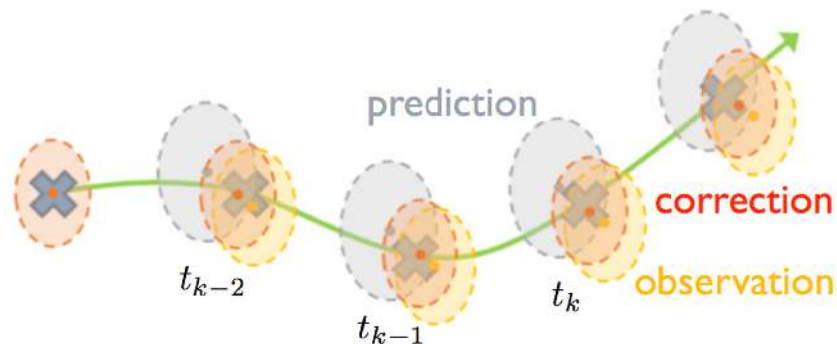
- ➡ modal signature is the key input feature for linear control
- ➡ on-the-fly assimilation to monitor frequency drop (due to damage)

Sequential Version of mCRE

■ Kalman Filtering

Dynamical system (linear): $x_k = M_k x_{k-1} + B_k u_k + w_k$ (model for state evolution)

$$y_k = H_k x_k + v_k \text{ (observations)}$$



recursive state estimate [Kalman 1960]
(MAP for Gaussian PDFs)

Prediction $\hat{x}_{k|k-1} = M_k \hat{x}_{k-1|k-1} + B_k u_k$ (mean)

$$P_{k|k-1} = M_k P_{k-1|k-1} M_k^T + Q_k \text{ (covariance)}$$

$\rightarrow E[w_k w_k^T]$

Correction $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - H_k \hat{x}_{k|k-1})$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

$$K_k = P_{k|k-1} H_k^T S_k^{-1} \text{ (Kalman gain: weights innovation)}$$

$$S_k = H_k P_{k|k-1} H_k^T + R_k \rightarrow E[v_k v_k^T]$$

Sequential Version of mCRE

■ Extension to nonlinear dynamical systems

$$\begin{aligned} x_k &= M(x_{k-1}, u_k) + w_k \\ y_k &= H(x_k) + v_k \end{aligned}$$



extended KF (EKF)
unscented KF (UKF) [Julier & Uhlmann 1997]
 ...

- deterministic selection of σ -points (Cholesky decomposition of cov. matrix)
- transformation of these points $\{x_i\}_{i=1,\dots,2N+1}$ through nonlinear functions (// comput.)
- high accuracy (order 3), but strongly depends on data noise [Li *et al.* 16]

■ Extension to parameter estimation

$$x_k = M(x_{k-1}, p_k, u_k) + w_k \longrightarrow \text{state + parameter estimation}$$

Joint Kalman Filter (KF on extended state)	Dual Kalman Filter (combines 2 KFs)
$p_k = p_{k-1} + w_k^p$ (stationarity hyp.) $\tilde{x}_k = [x_k^T, p_k^T]^T$ (concatenation)	$p_k = p_{k-1} + w_k^p$ (parameters as state vect.) $y_k = H_{\text{dual}}(x_k, p_k, u_k, w_k) + v_k$ $\uparrow$ observer = state evaluation operator (based on a second KF, for given p_k)



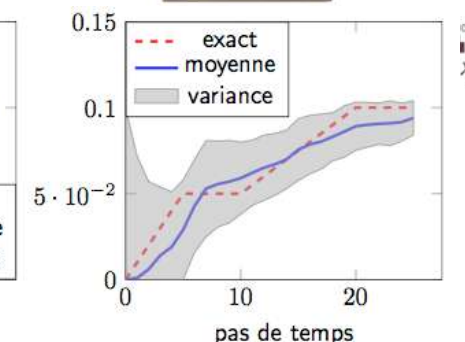
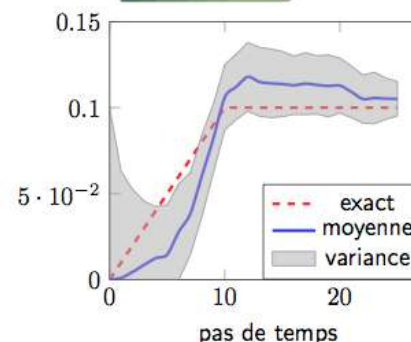
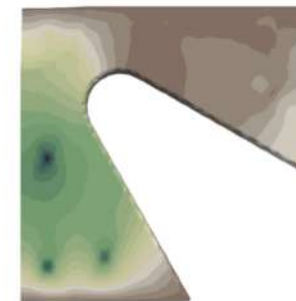
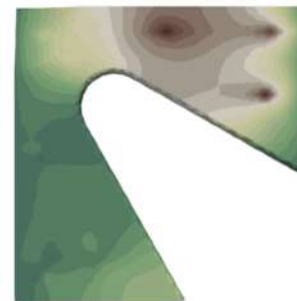
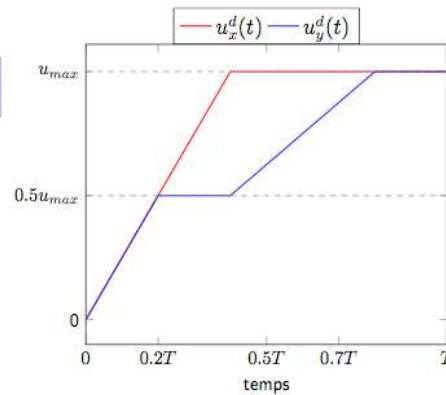
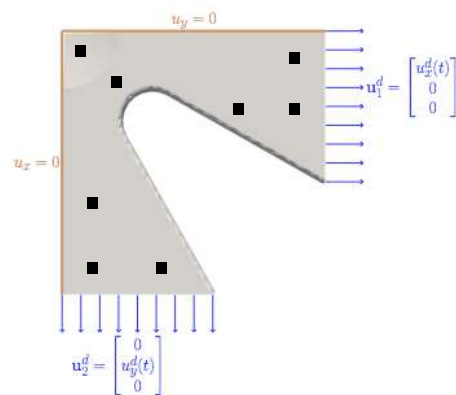
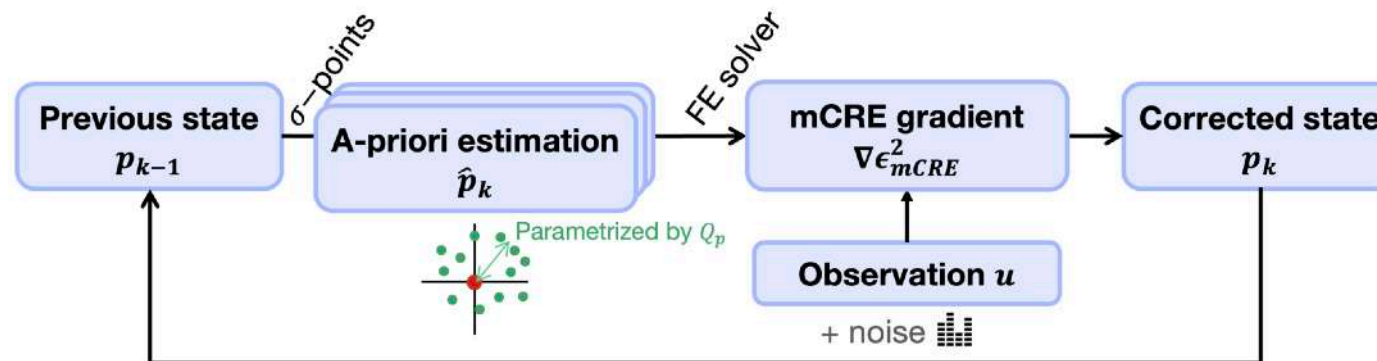
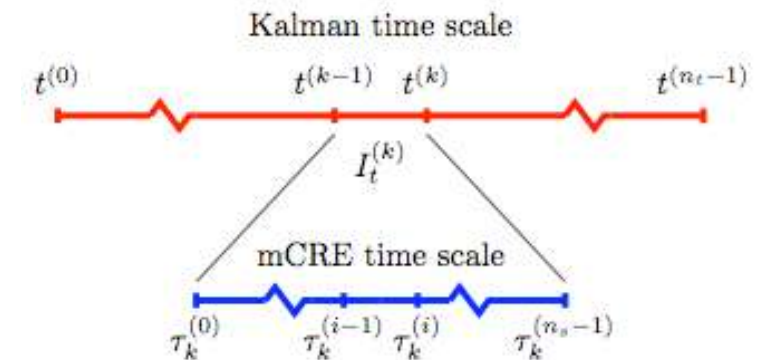
change of observer metric using mCRE: **MDKF strategy (enhanced robustness to noise)**

$$\nabla_p \mathcal{E}_{mCRE}(x_k, p_k, y_k) + v_k = 0 \quad [\text{Marchand } et al. 2016, \text{ Diaz } et al. 2022]$$

Sequential Version of mCRE

■ Modified Dual Kalman Filter (MDKF) [Marchand *et al.* 16, Diaz *et al.* 2022,2023]

- ➔ change of observer metric using mCRE in Dual KF
- ➔ considers optimal admissible fields as state estimate
- ➔ automatic calibration of MDKF internal parameters
- ➔ enhanced robustness to measurement noise compared to classical UKF
- ➔ use of ROM and other advanced techniques for further numerical efficiency



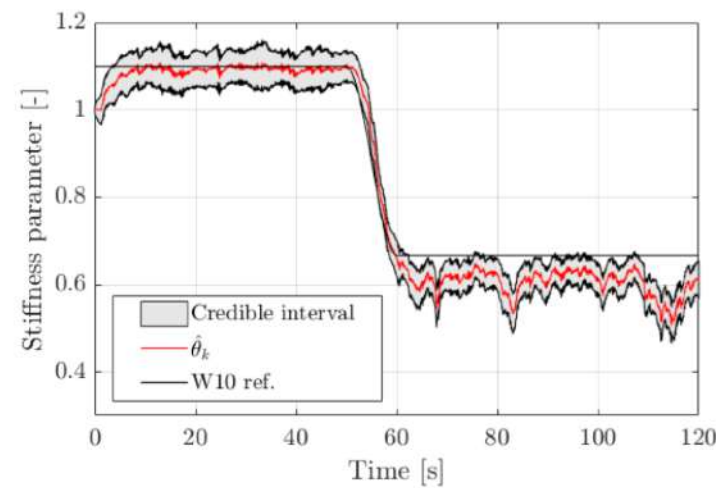
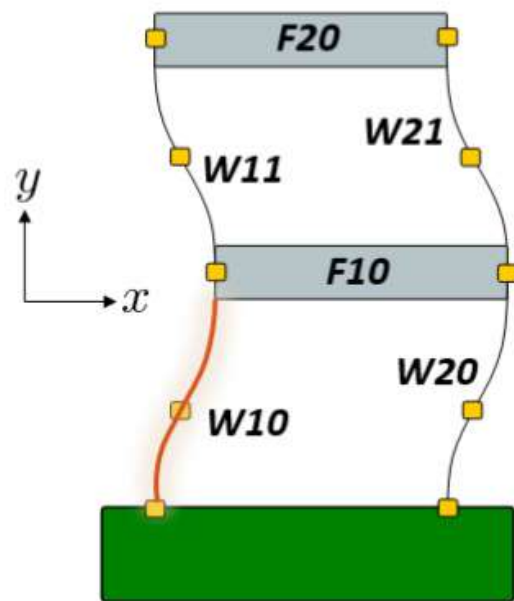
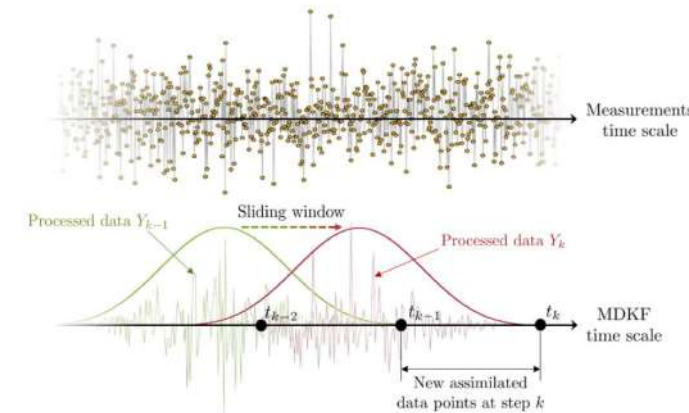
average CPU time
 $\approx$
 0.2s



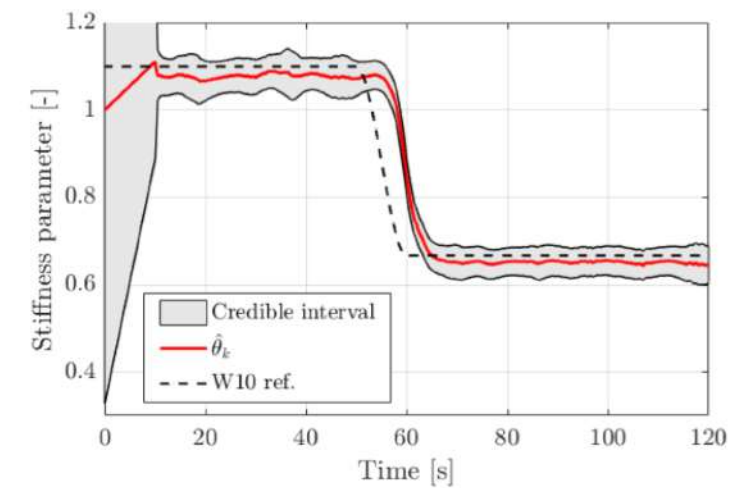
Sequential damage assessment in dynamics

mCRE in the frequency domain

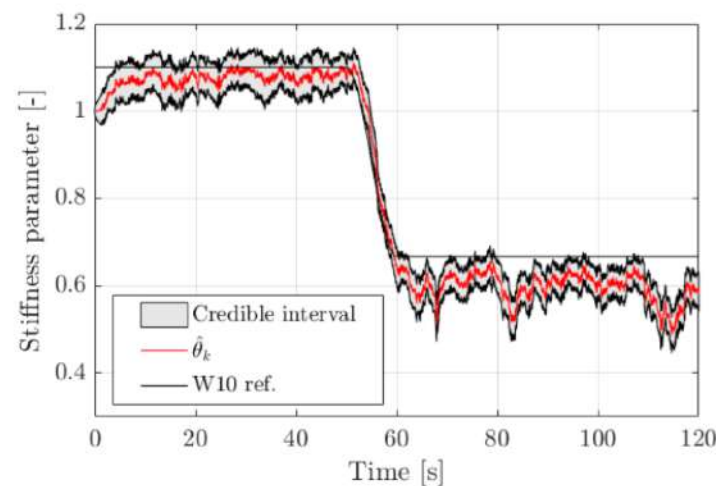
$$e_{\omega}^2(s, p, Y_{\omega}) = \zeta_{\omega}^2(s, p, Y_{\omega}) + \alpha \|\Pi \circ s - Y_{\omega}\|_G^2 \quad \Rightarrow \quad J(p) = \int_{D_{\omega}} z(\omega) e_{\omega}^2(\hat{s}(p), p, Y_{\omega}) d\omega$$



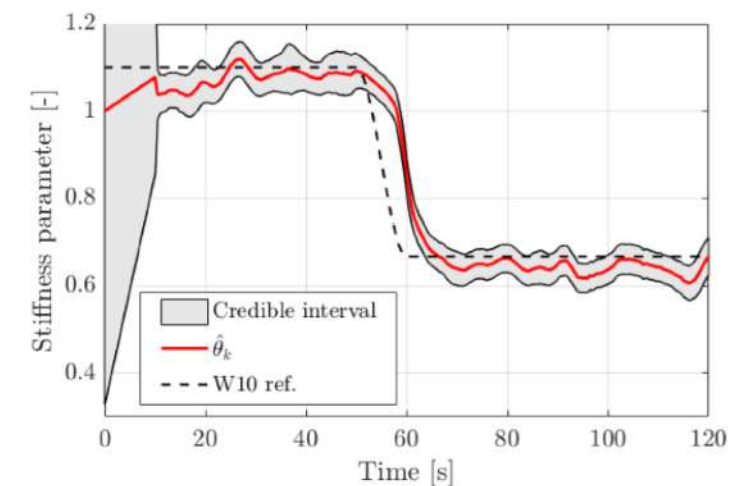
(a) JUKF - $\delta = 0\%$, $Q_x = 10^{-8}I$, $Q_{\theta} = 10^{-7}I$, $R = 10^{-2}I$



(b) MDKF - $\delta = 0\%$, $r = 0.5$



(c) JUKF - $\delta = 20\%$, $Q_x = 10^{-8}I$, $Q_{\theta} = 10^{-7}I$, $R = 10^{-2}I$



(d) MDKF - $\delta = 20\%$, $r = 0.001$

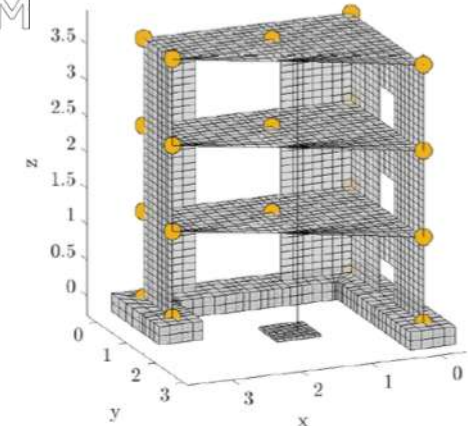
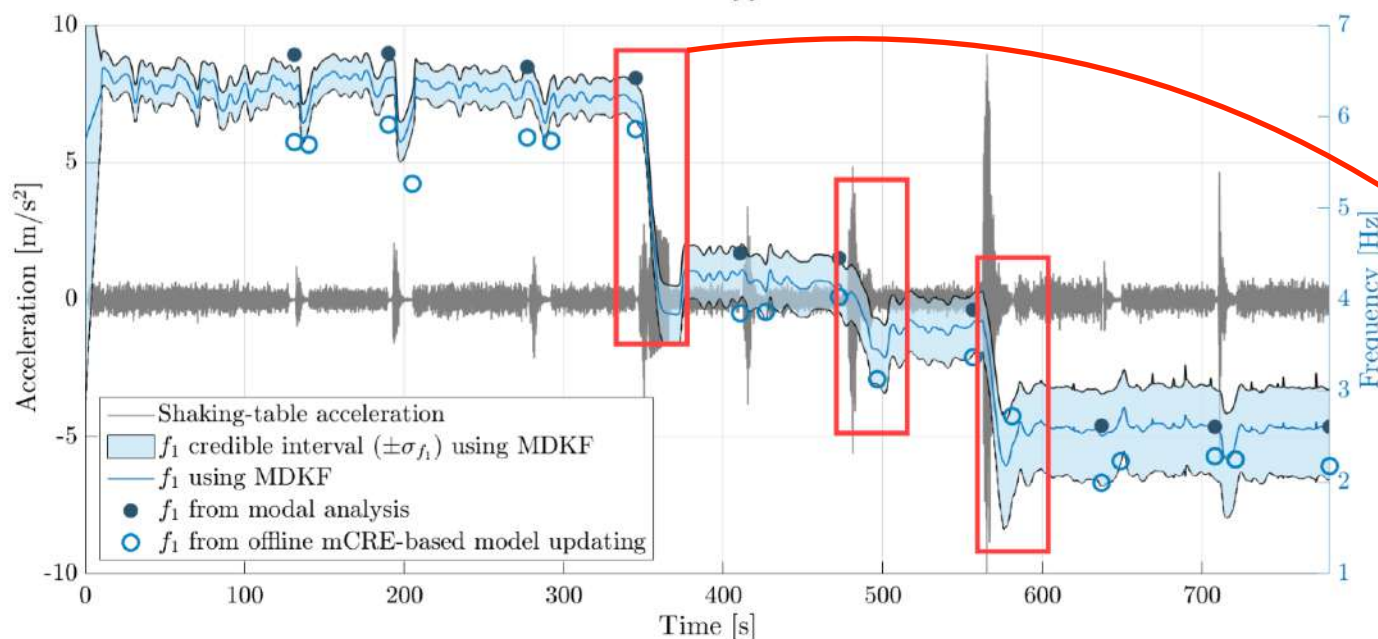
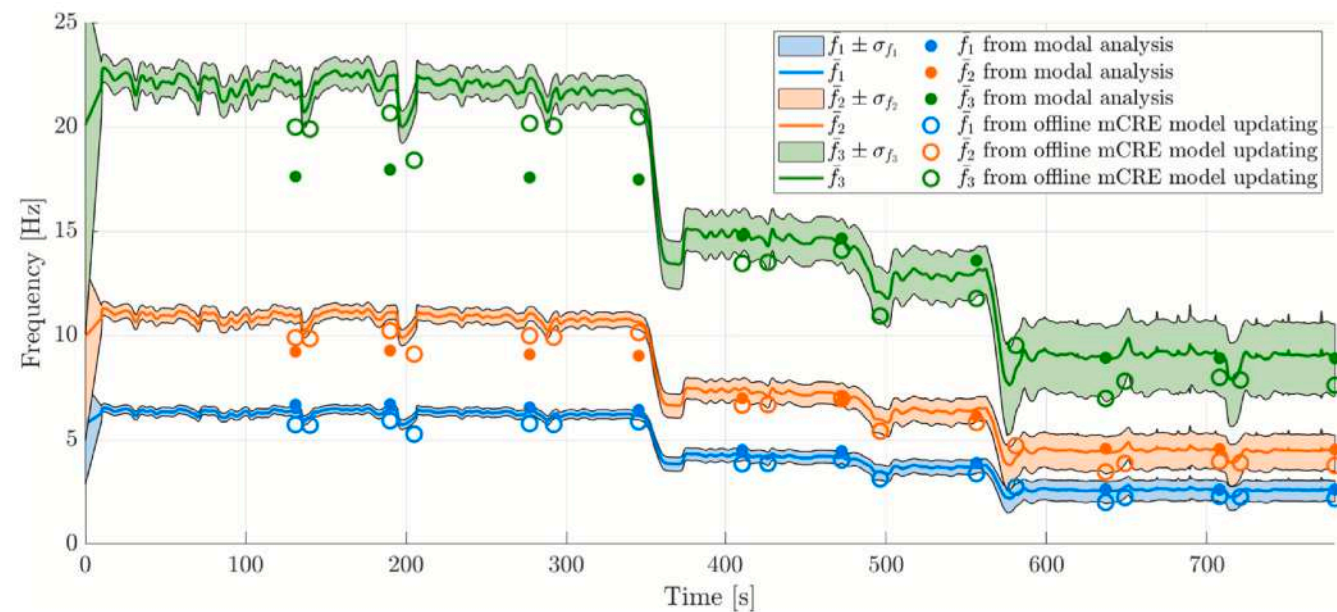
■ Eigenfrequency tracking from sparse & highly noisy measurements

- results obtained with 41 accelerometer recordings from the campaign
- distributed damage parameters on walls & floors
- tracking of the 3 first eigenfrequencies
- real-time constraint successfully achieved

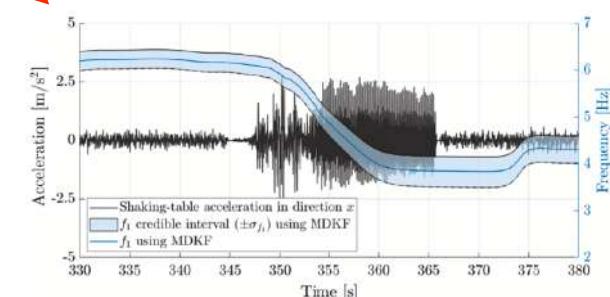
data window = 0.5 s
assimilation < 0.3 s



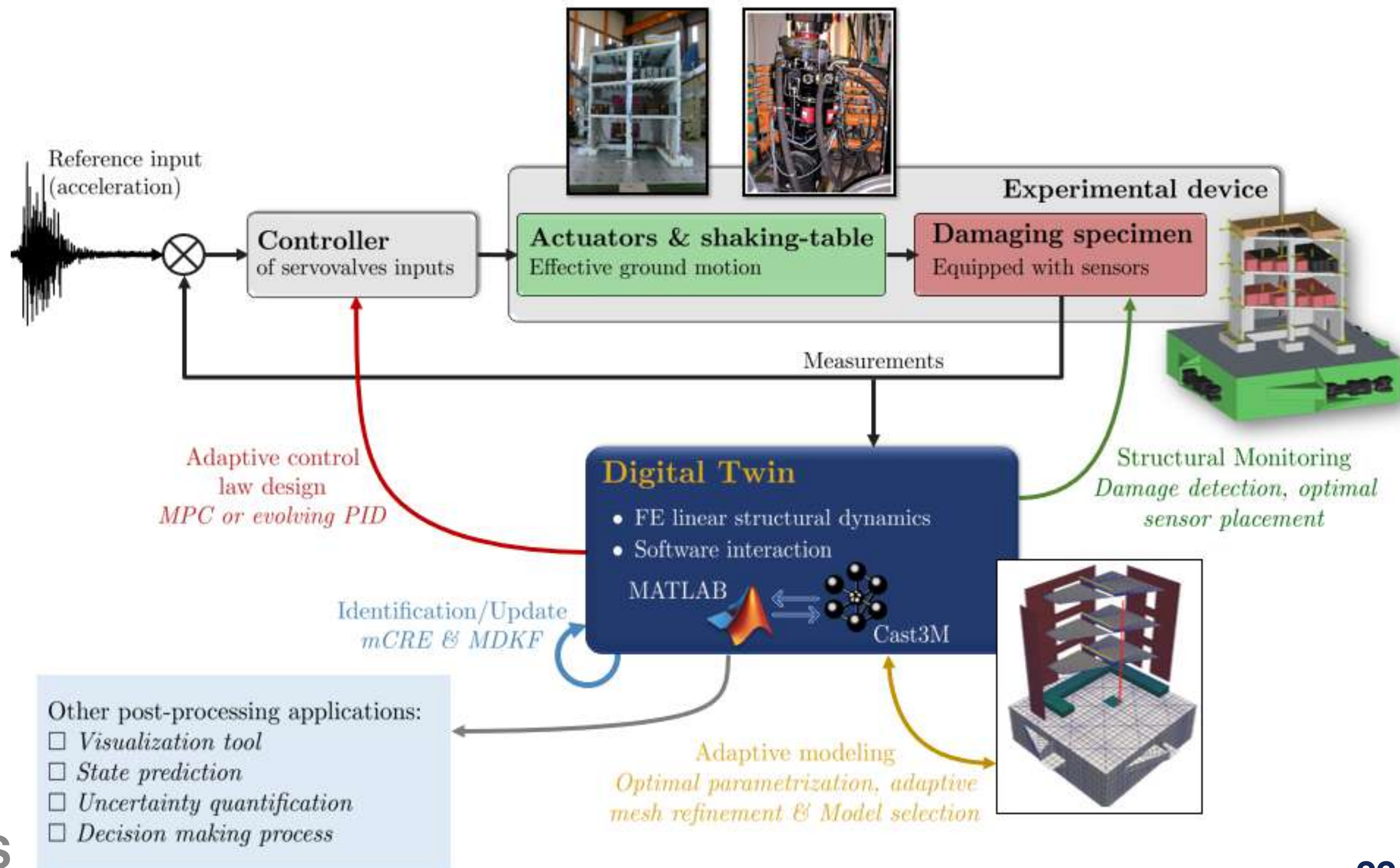
RC specimen of SMART2013 anchored on the AZALEE shaking-table



FE model from CEA-EMSI developed in Cast3M

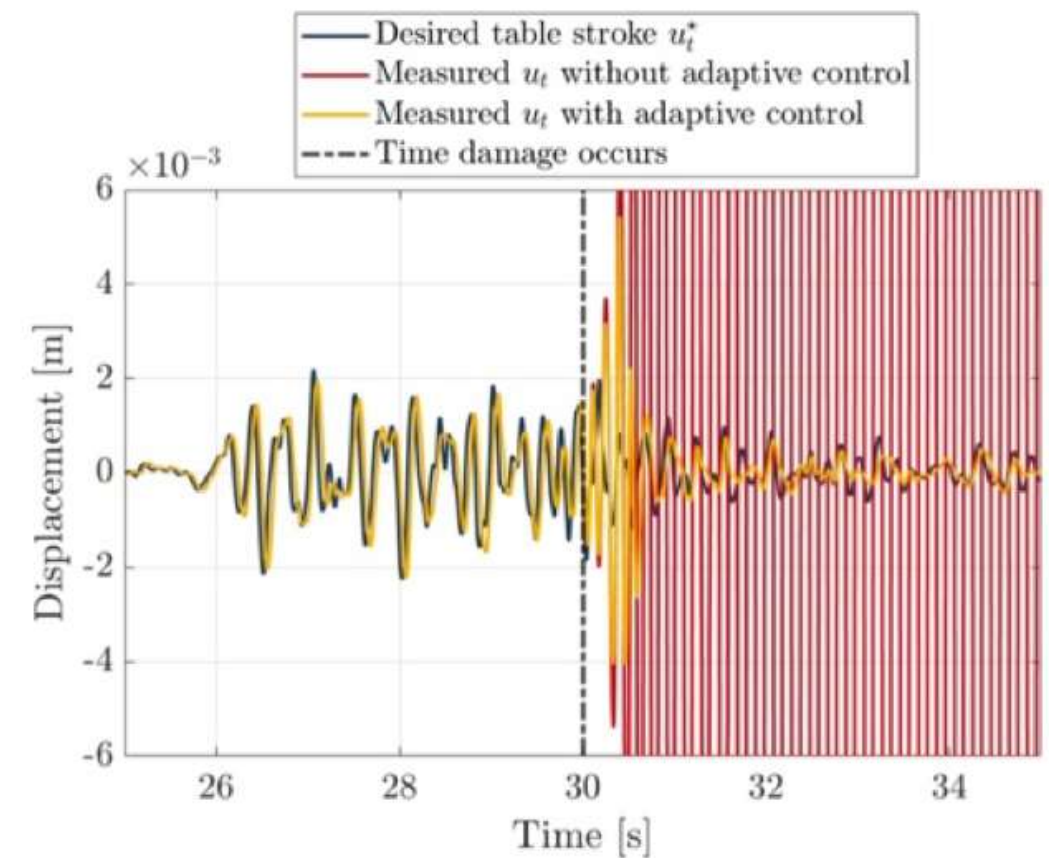
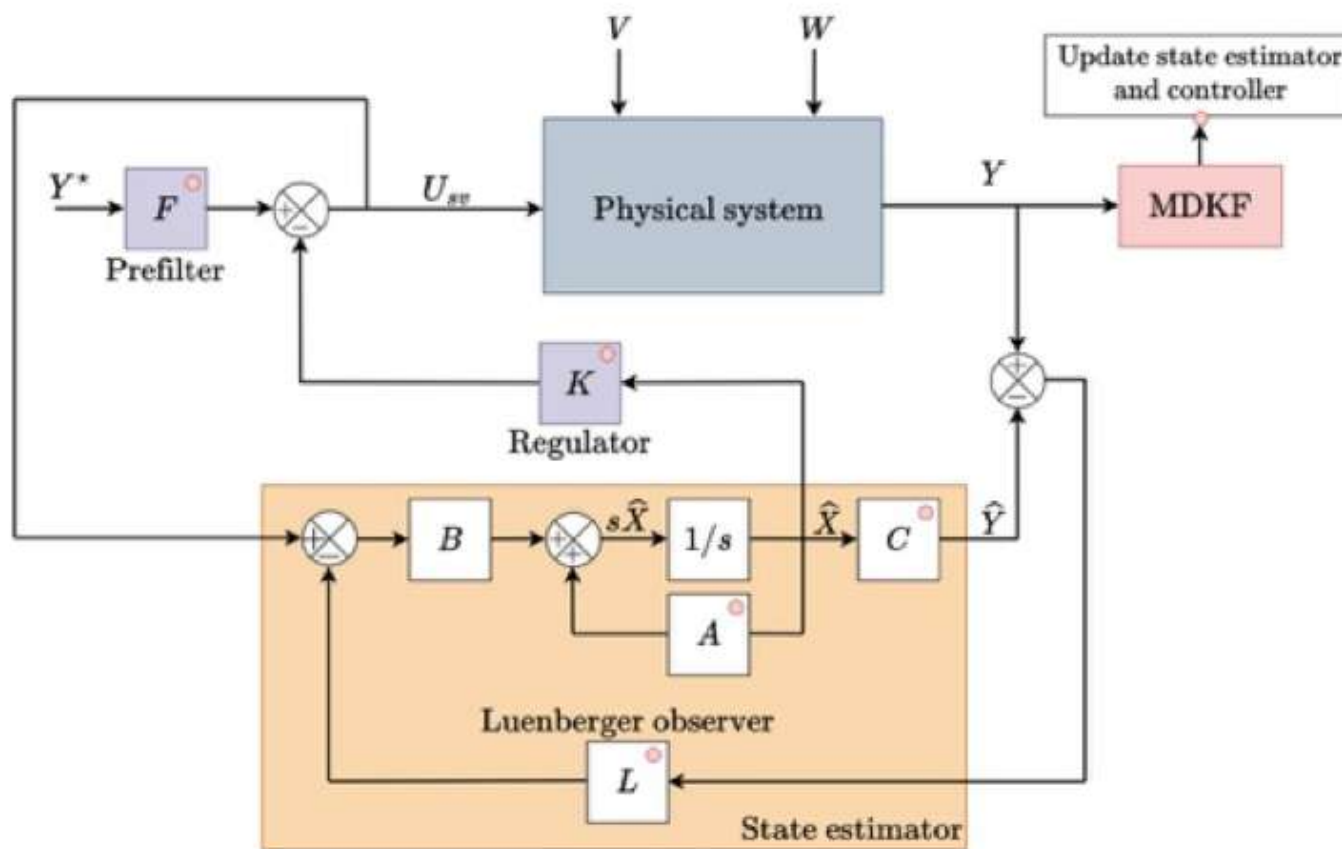
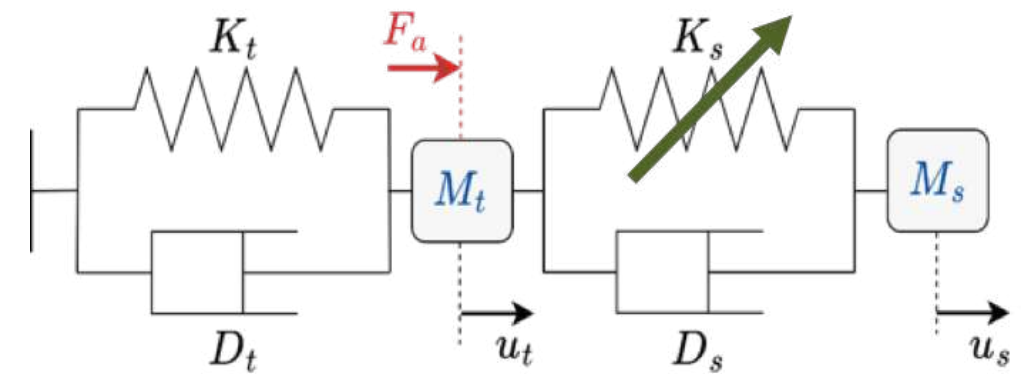


■ On-the-fly control with adaptive command synthesis



■ On-the-fly control with adaptive command synthesis

- automatic dynamical drive of hydraulic actuators
- MDKF strategy inserted in a LQP controller



- **Illustrated introduction to hybrid twins**
- **mCRE framework for data assimilation**
- **Sequential strategy coupling mCRE with Kalman filtering**
Application to the online control of shaking table tests on large-scale damageable concrete buildings
- **On-the-fly adaptive modeling for enhanced computational efficiency**
Application to the design of smart structures equipped with embedded distributed optic fiber sensors



DREAM-ON Project

[ERC-CoG 2021-2026]



<https://erc-dreamon.ens-paris-saclay.fr>

➡ SMART ENGINEERING STRUCTURES

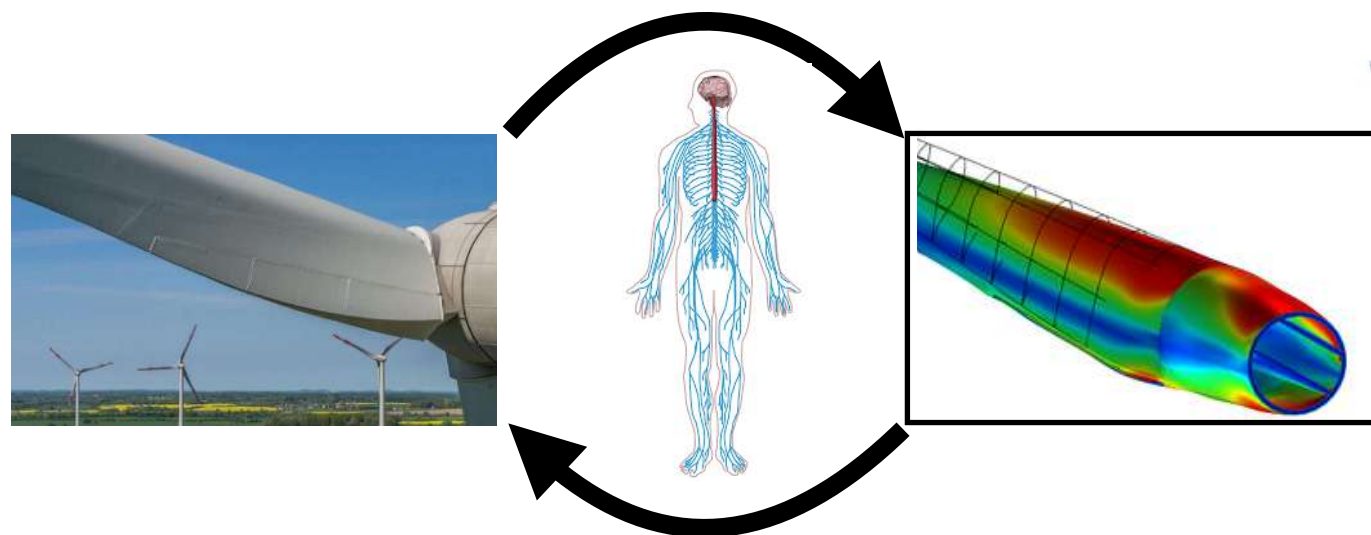
- Integrated simulation-based SHM ➡ online monitoring of structural integrity
- Damage diagnosis, prognosis, and control

Enhanced durability and performance

Anticipated action during service

Optimized maintenance

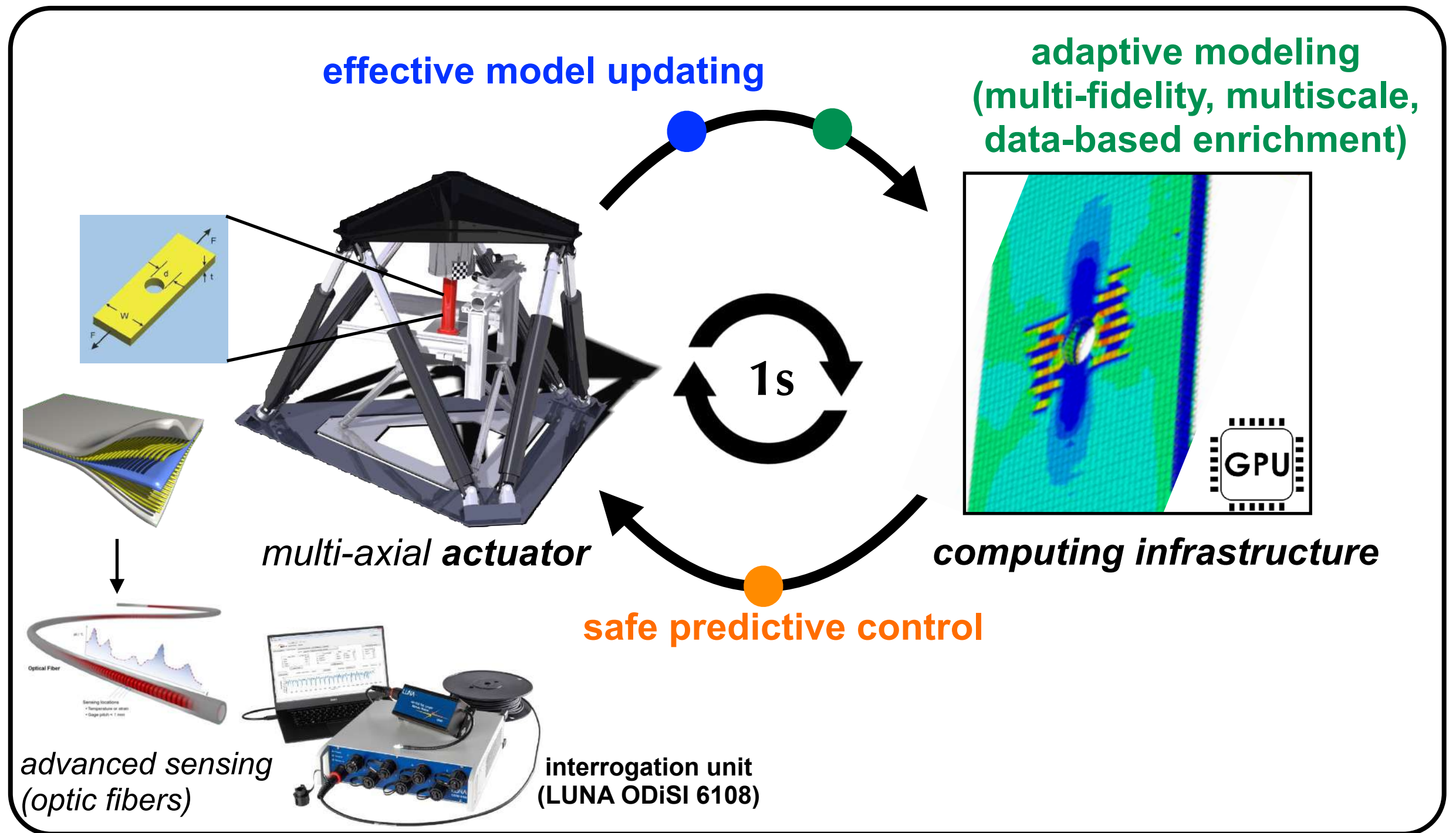
Operation in degraded mode



- Advanced sensing + HF modeling + powerful numerical methods
- Constraints: real-time, portability (edge computing), robustness

DREAM-ON: merging advanced sensing techniques and simulation tools for future SHM technologies,
The Project Repository Journal, 10:124-127 (2021)

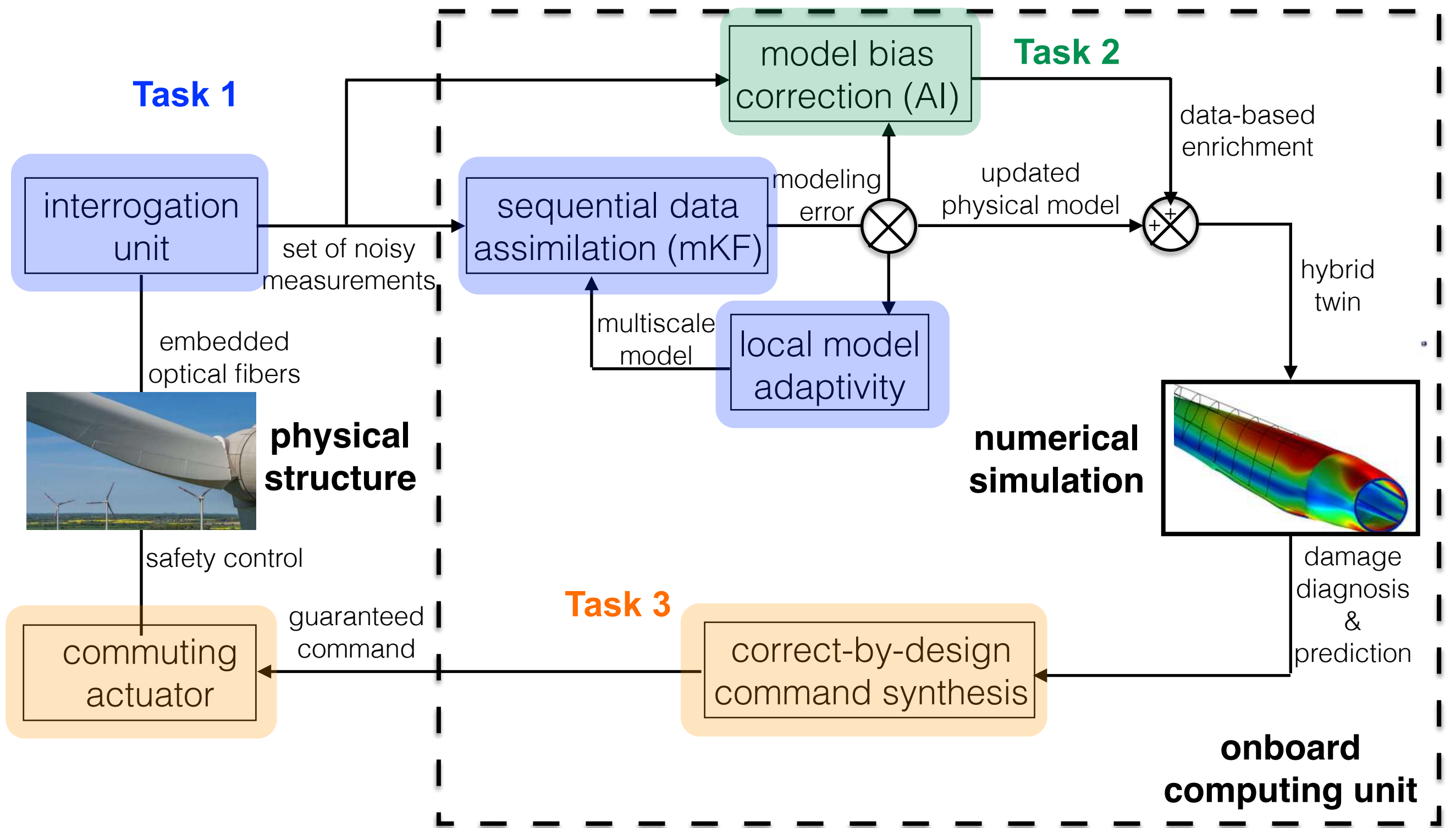
Proof-of-Concept



→ find an optimal loading path given different initial damage conditions

→ make the structure remain in a safety region (operation at performance limits) for various instances of damage

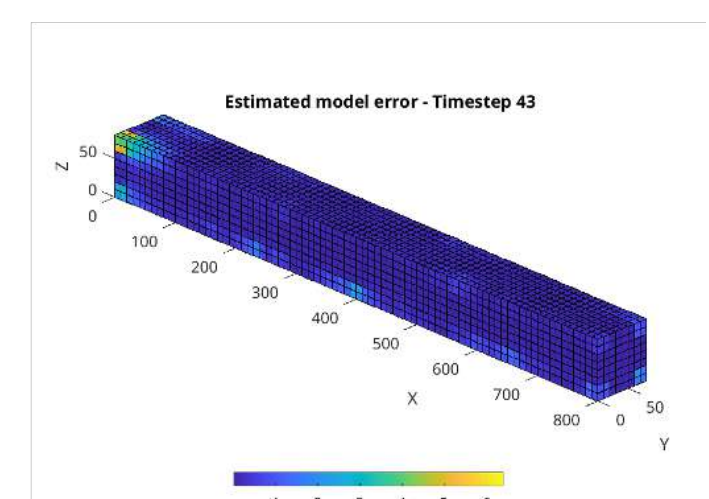
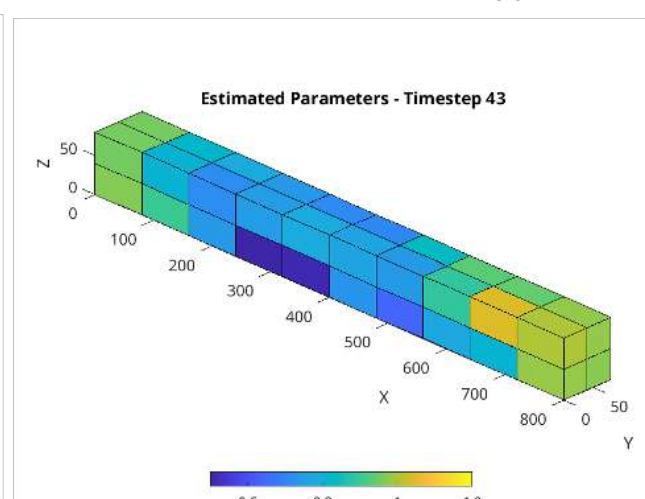
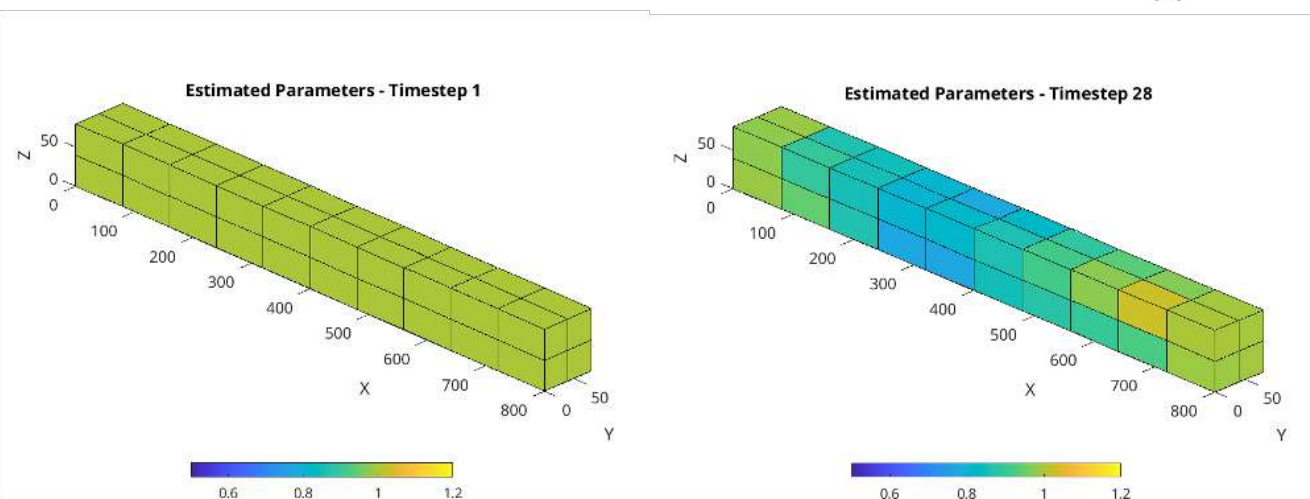
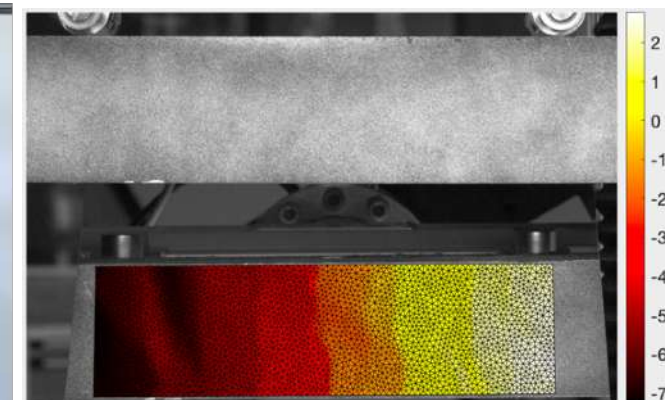
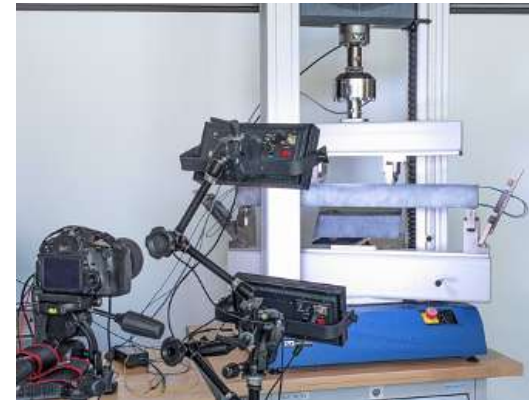
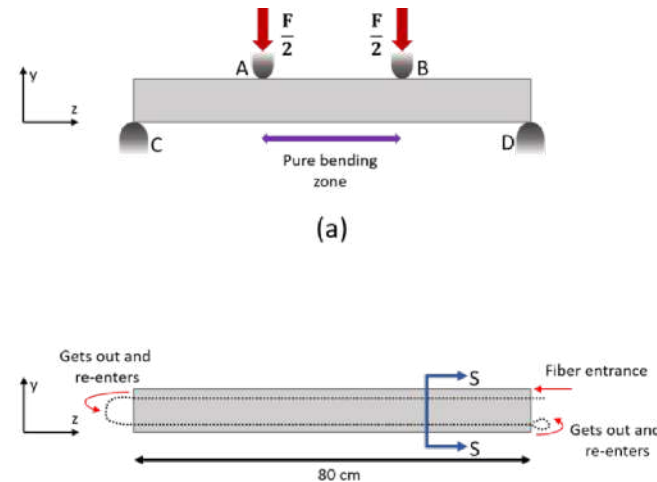
Global Strategy



L. C., E. Baranger, A. Benady, P-E. Charbonnel, M. Diaz, S. Farahbakhsh, L. Fribourg, D. Martin-Xavier, M. Poncelet, A novel DDDAS architecture combining advanced sensing and simulation technologies for effective real-time SHM, *Handbook of DDDAS*, Springer (2025)

Task 1: MDKF Strategy with DOFS

- **Beam samples under 4-point bending**
 - Cement-based mortar reinforced with polyamide and steel fibers
 - Linear, elastic, isotropic and homogeneous material
 - Updated parameter: Damage parameter in zones
 - Dimensions 80 x 8 x 8 cm
 - C3D8 elements (1 x 1 x 1 cm)
- **4 lines of measurements provided by optic fibers**
 - Measurement every 2.6 mm



- Application with complex anisotropic damage models (composites, concrete,...)
- Continuous model updating (along the structure life) from data obtained on-the-fly
- Outputs: updated model + mechanical state (optimal admissible fields)

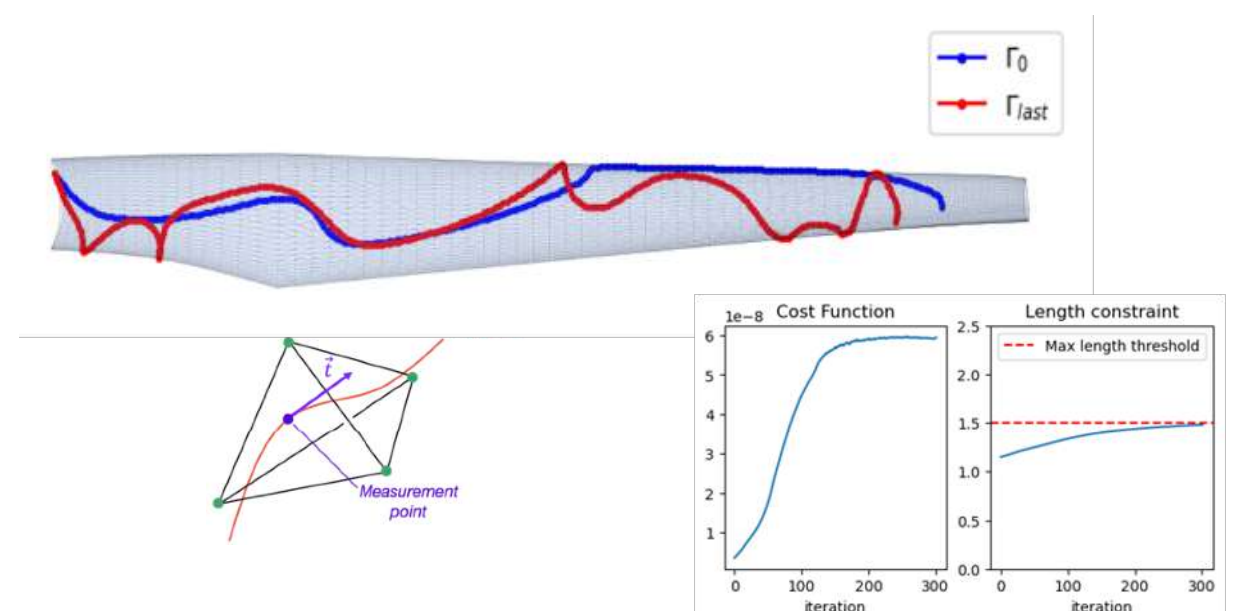
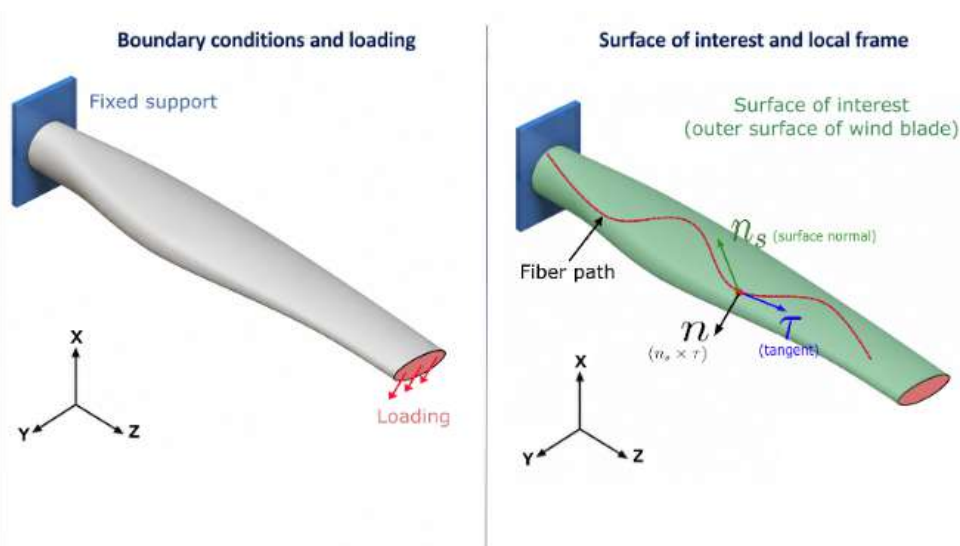
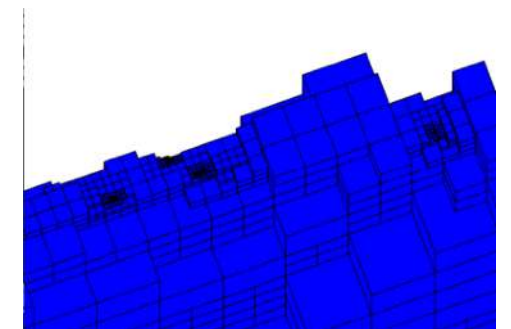
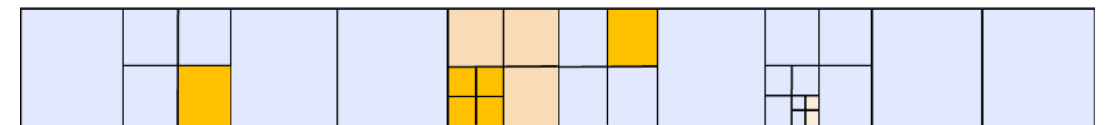
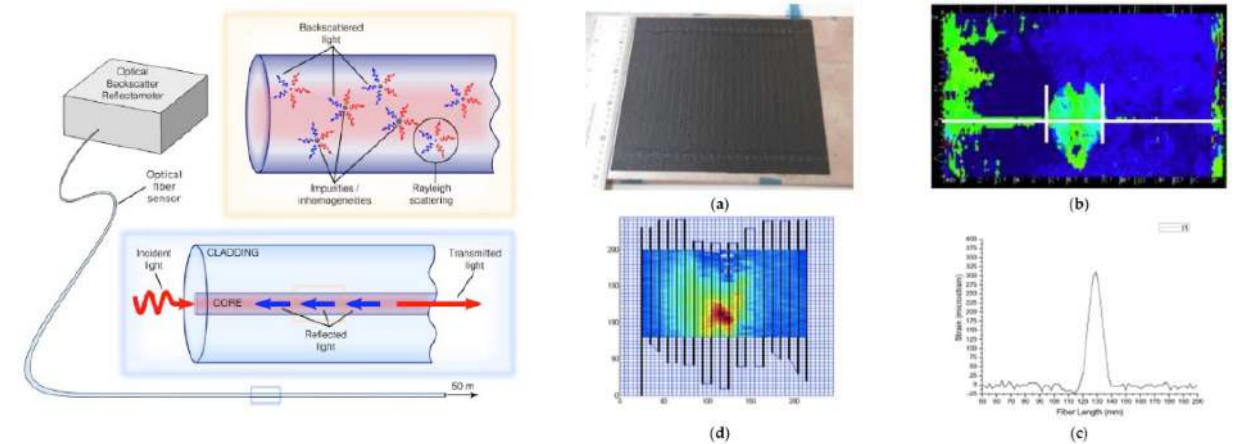
S. Farahbakhsh, M. Poncelet, L.C, Investigation of Distributed Optic Fiber Sensors data loss due to strain gradient, *Experimental Techniques*, online (2025)

S. Farahbakhsh, L.C., M. Poncelet, Continuous structural health monitoring with a modified dual Kalman filter applied with optic fiber sensing data, *Computers & Structures* (2026)

Task 1: MDKF Strategy with DOFS

■ Numerical optimizations

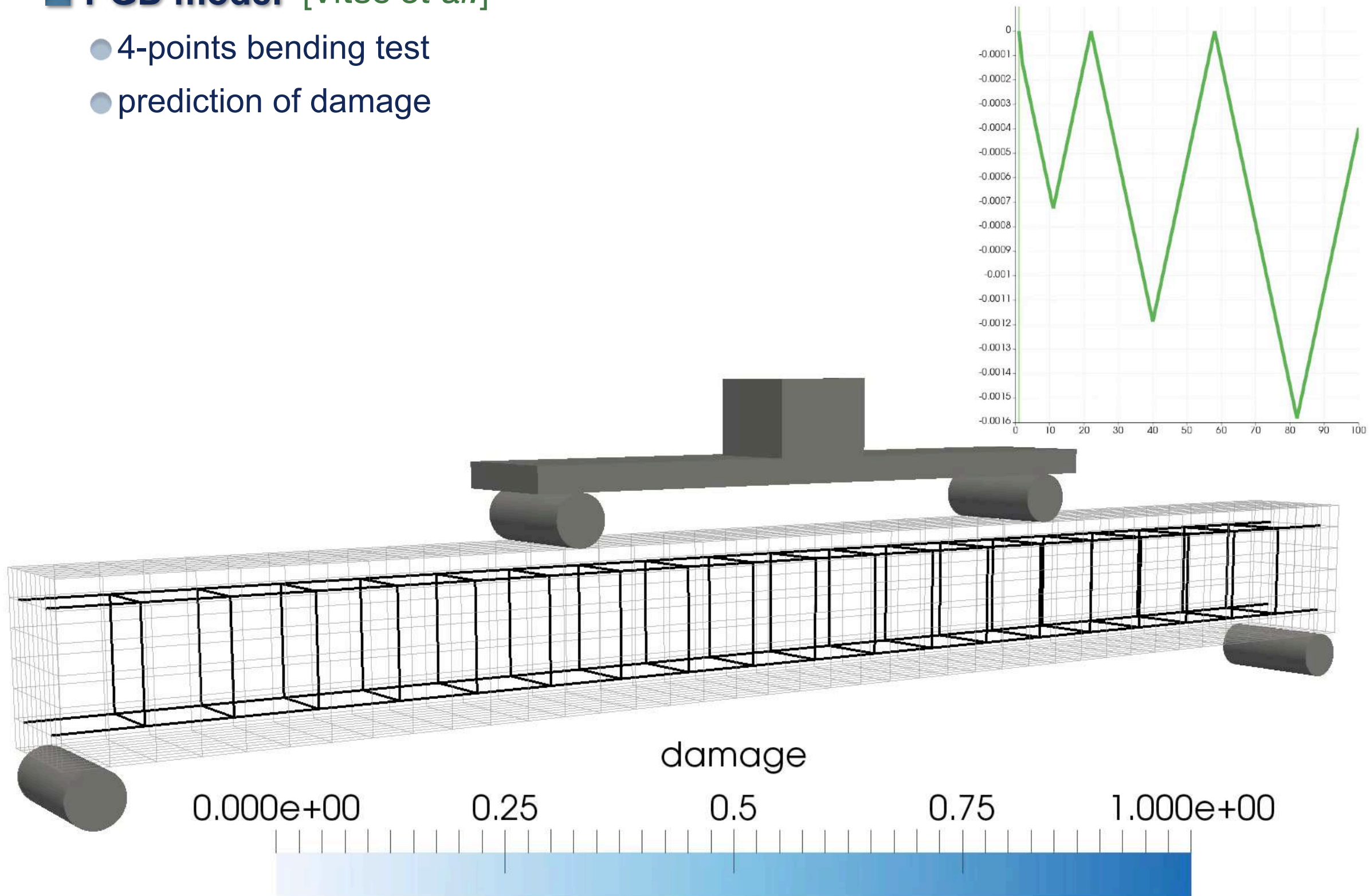
- filtering of DOFS data [Farahbakhsh *et al.* 2025]
- sparse regularization
- dynamical **adaptive modeling** in the parameter space (CgFEM)
- optimal DOF **sensor placement** [Perez Orozco *et al.* 2025]
(*B-Splines, Fisher information matrix, shape derivative, etc.*)

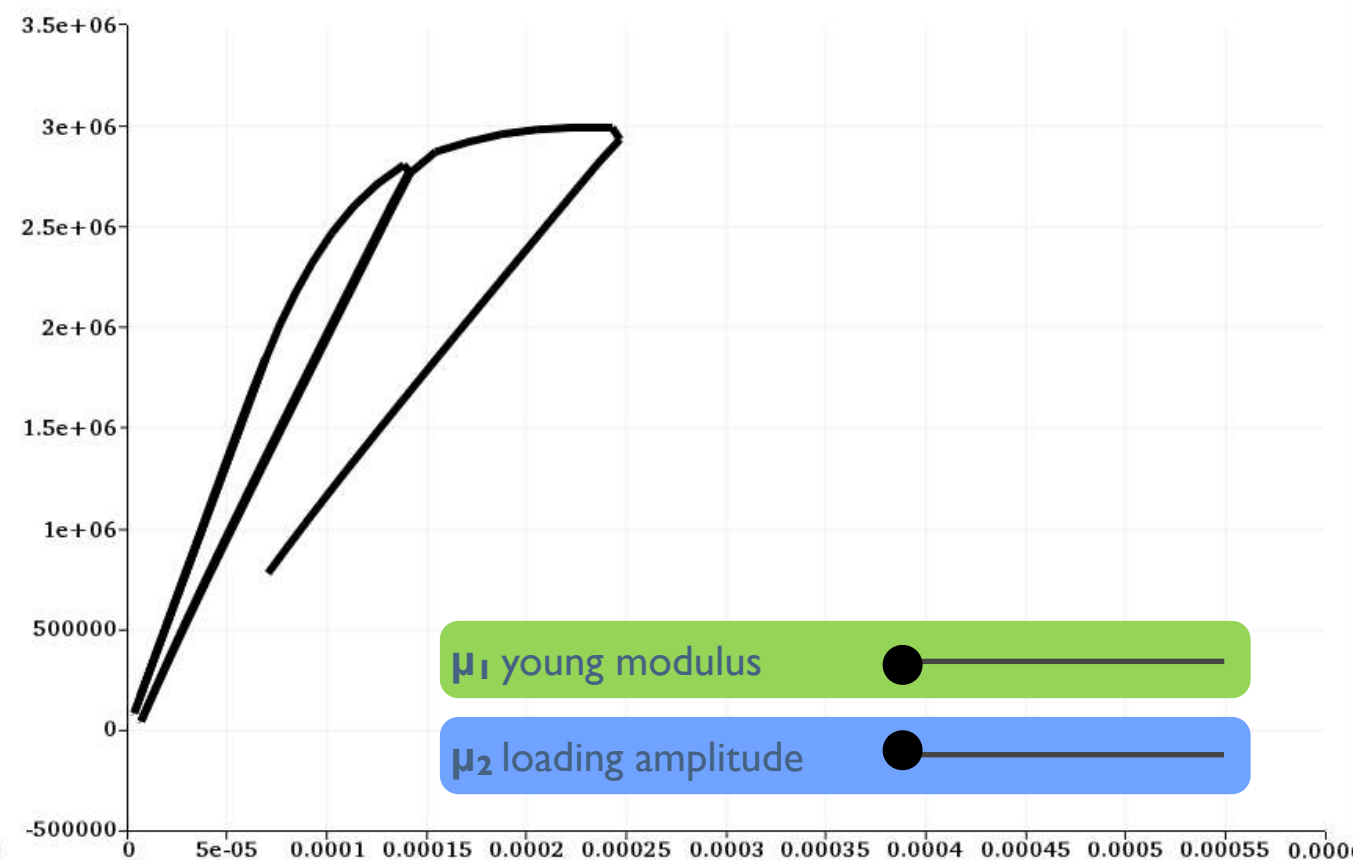
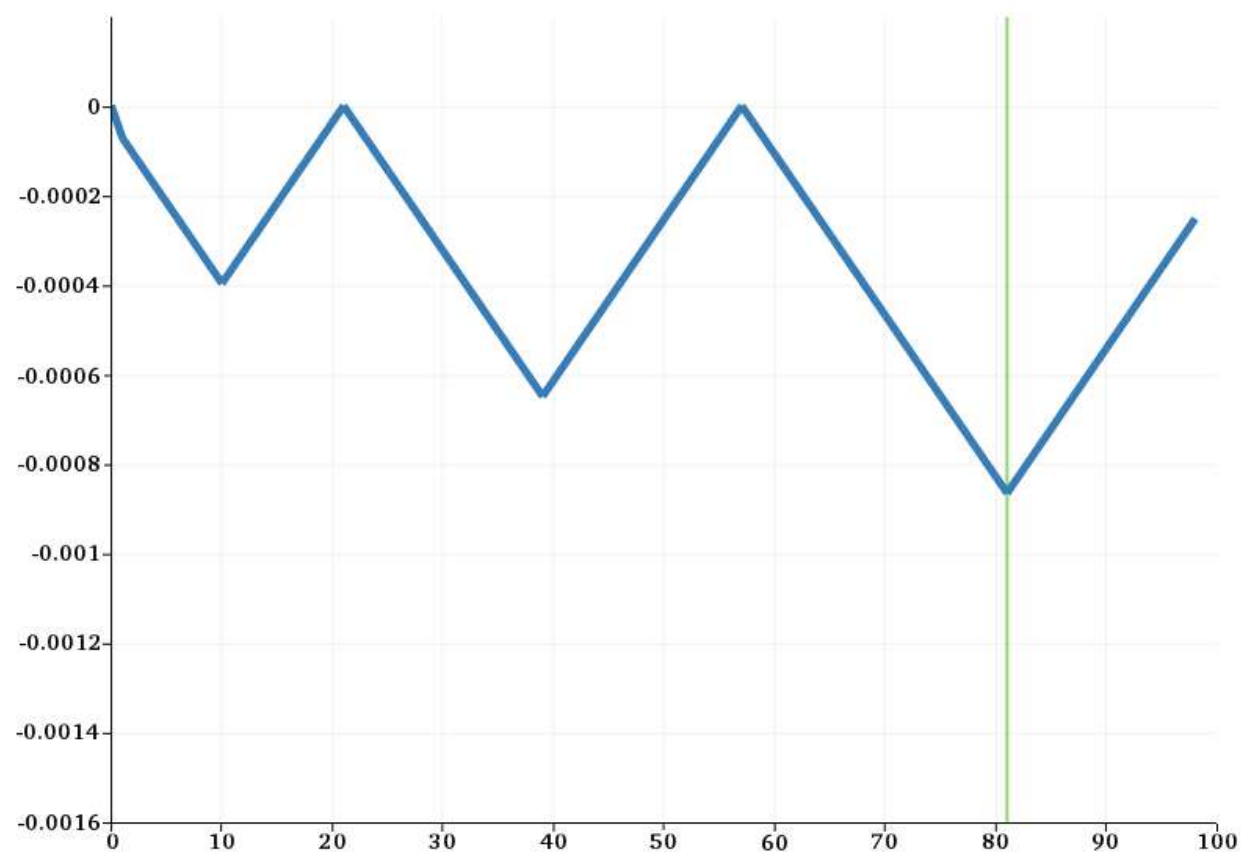
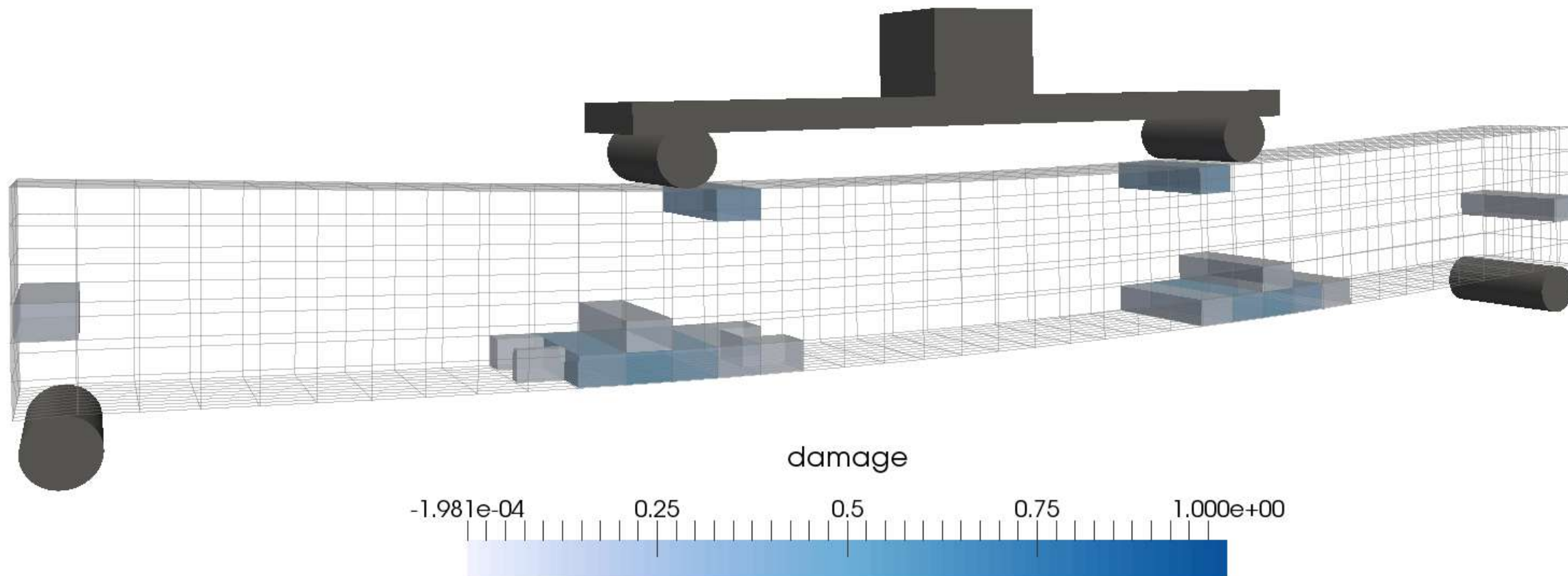


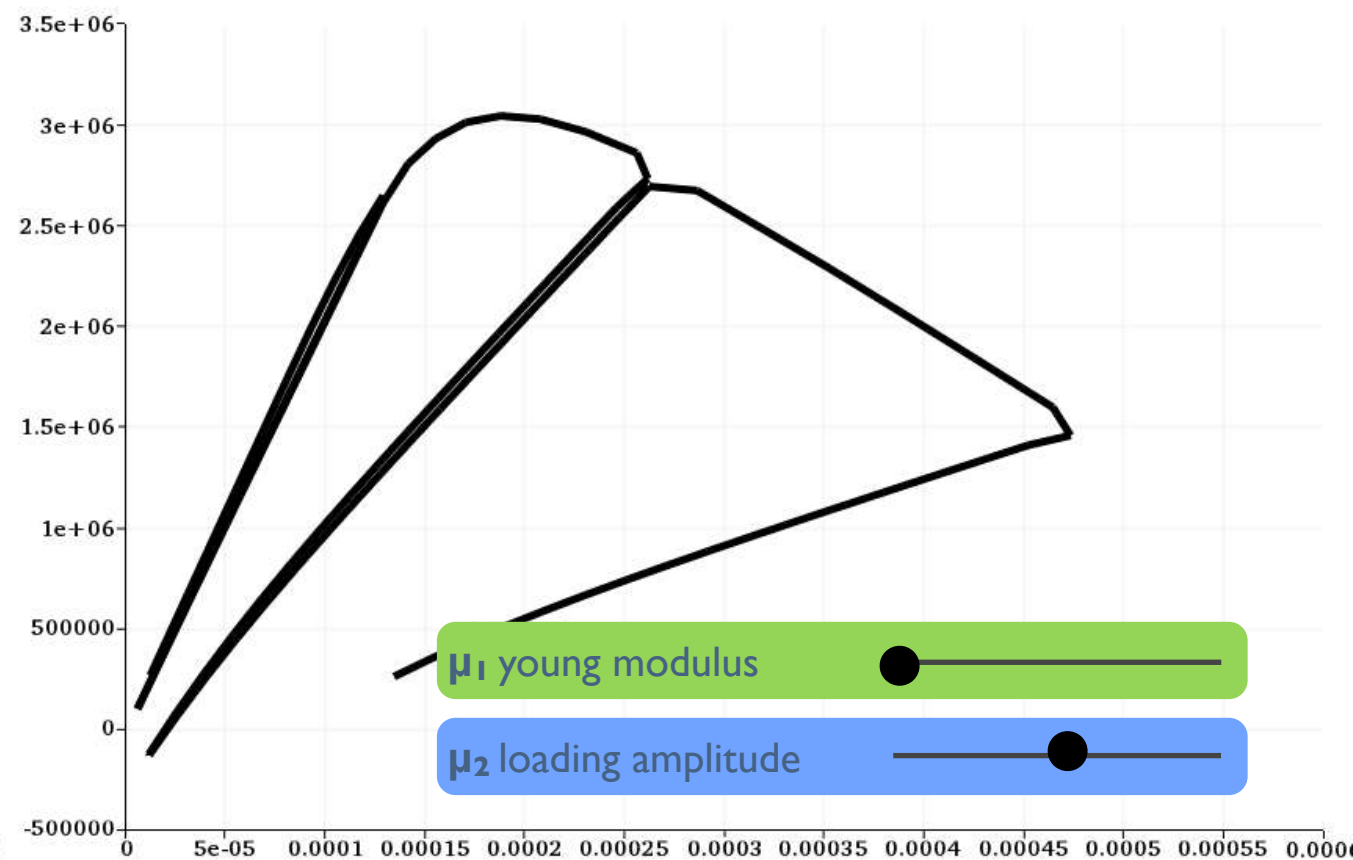
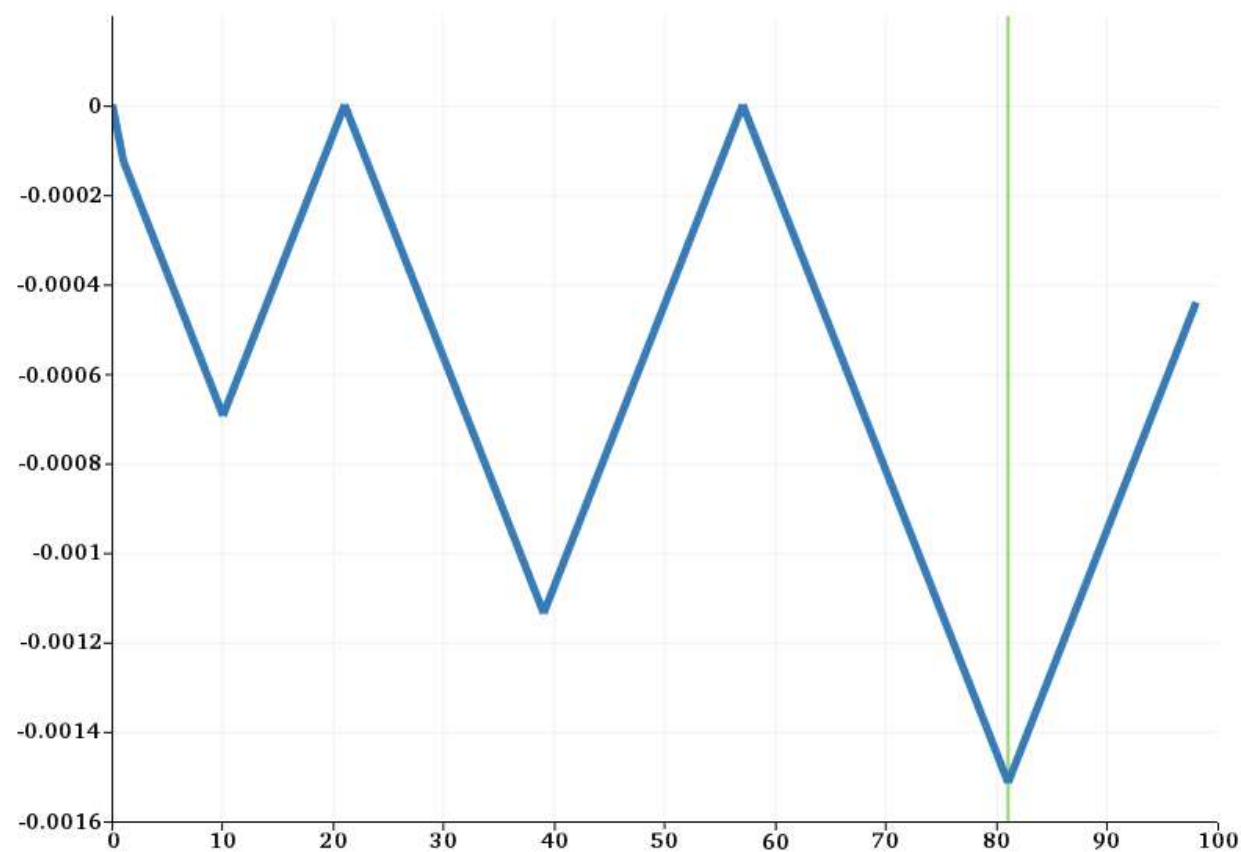
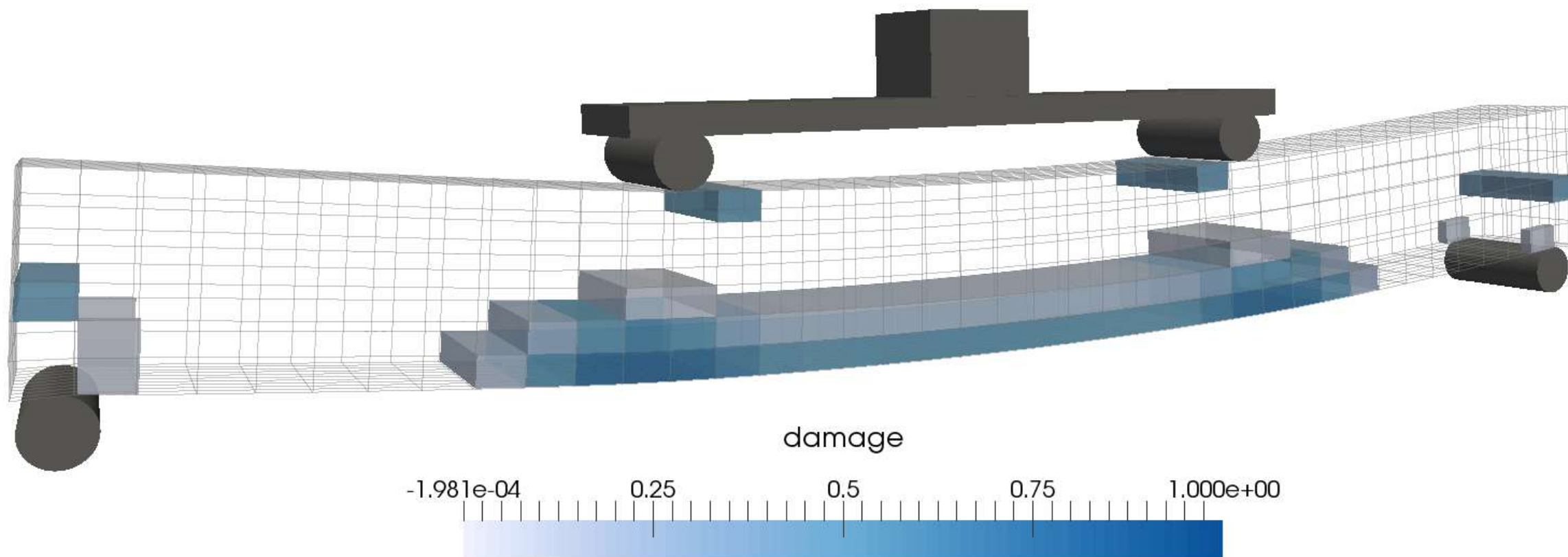
Task 1: MDKF Strategy with DOFS

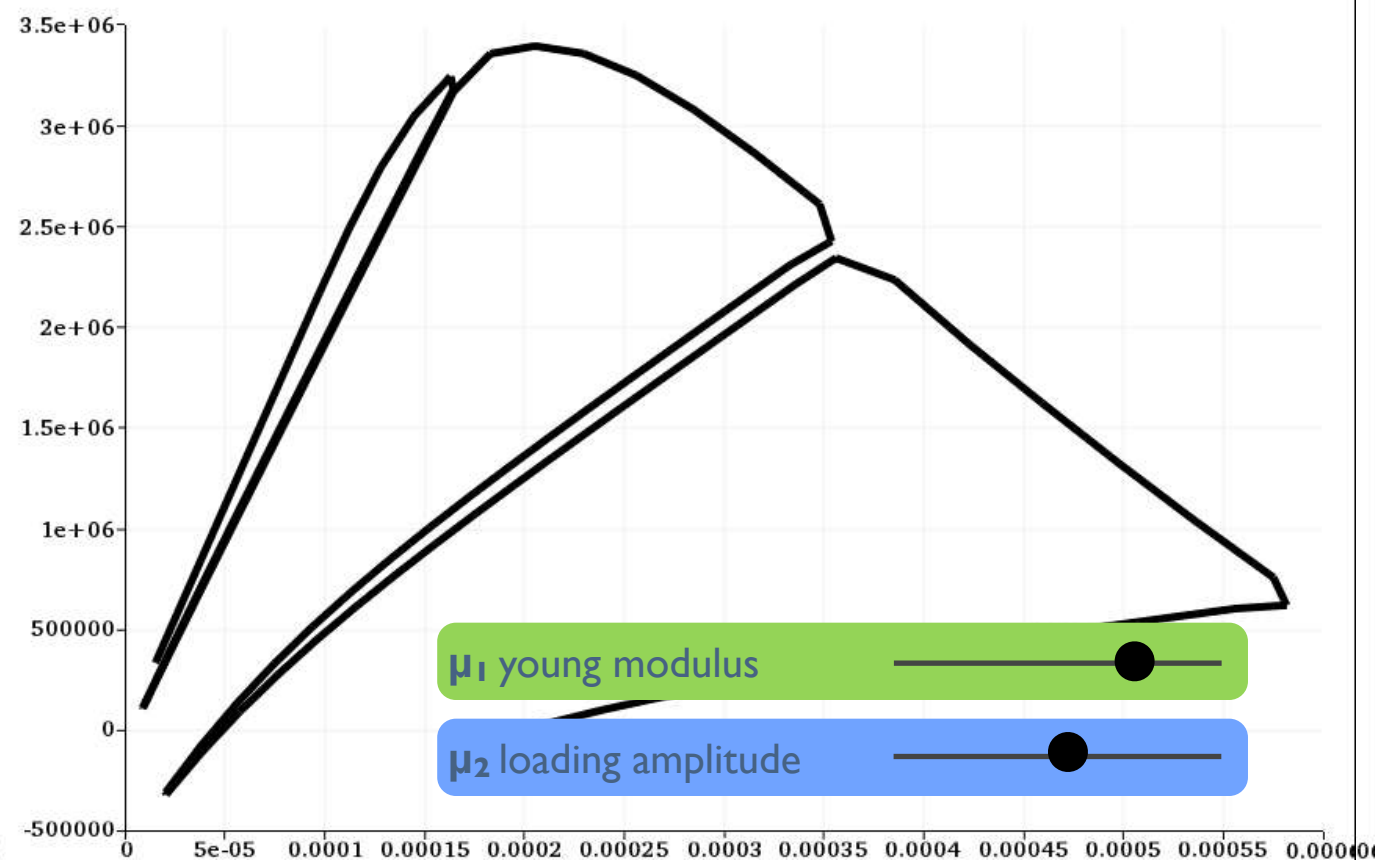
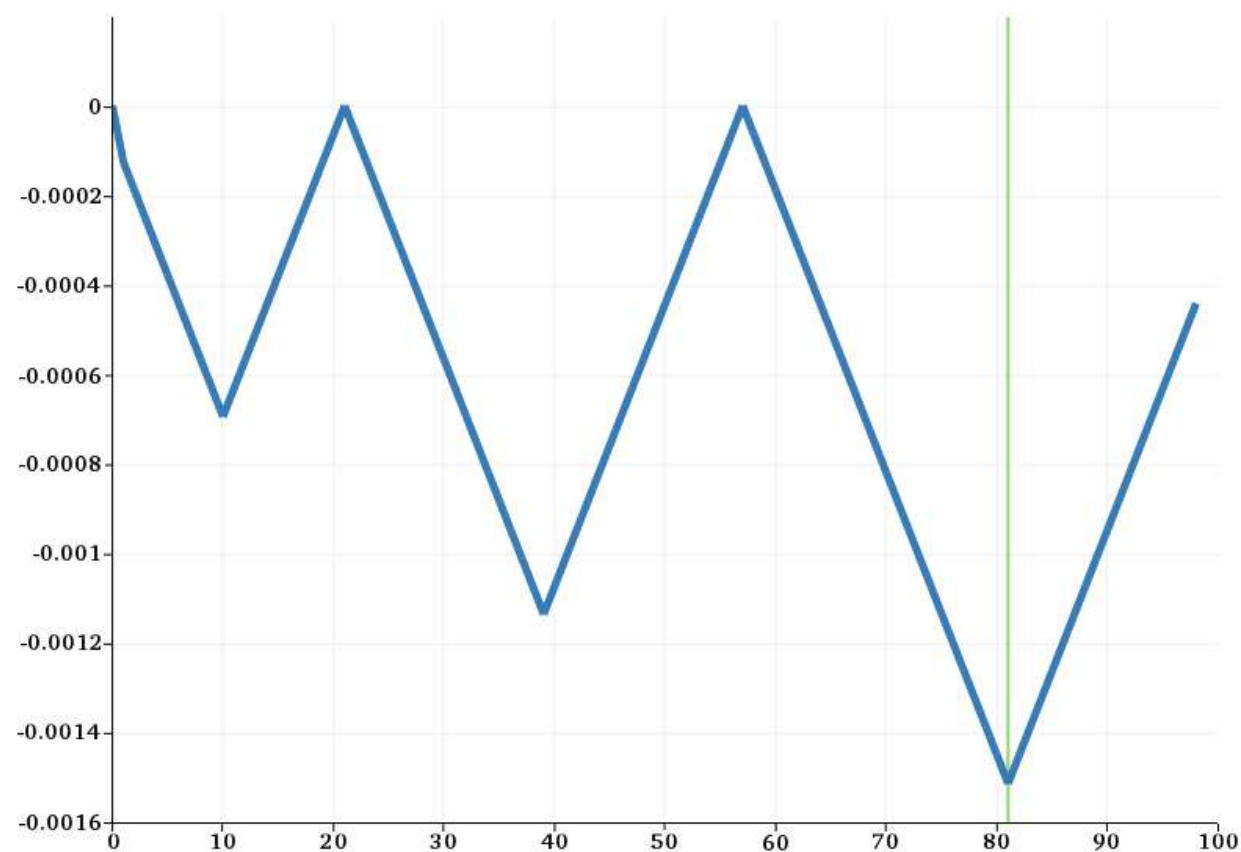
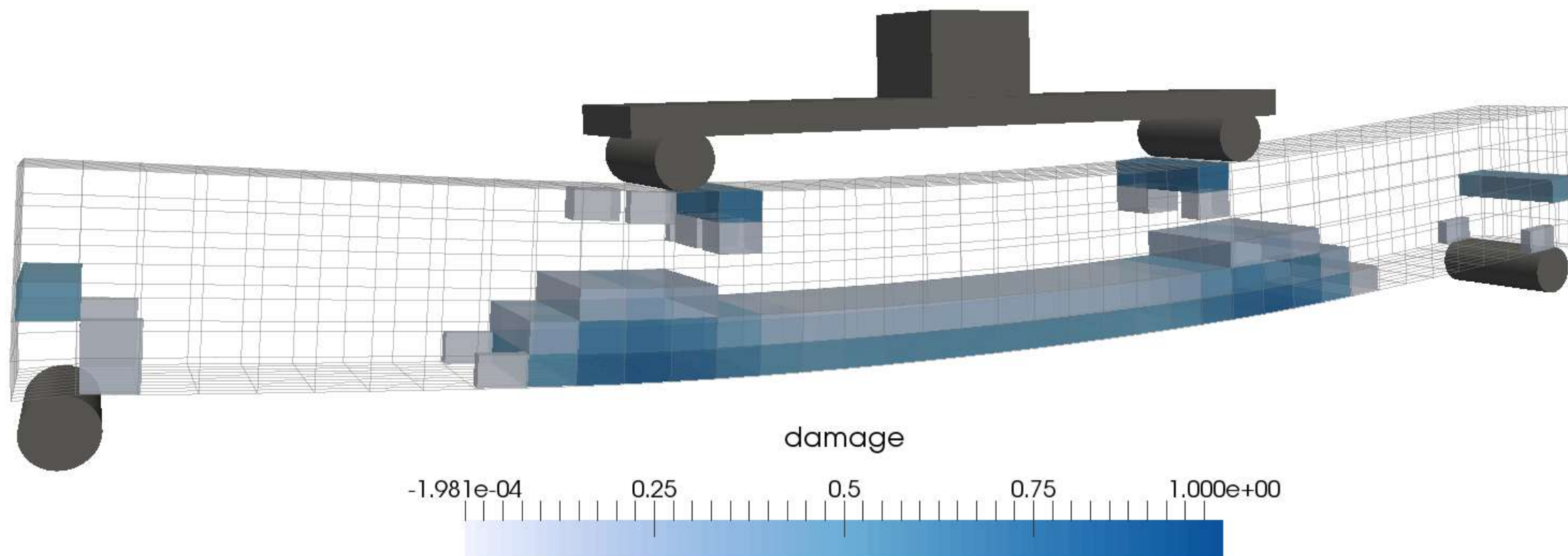
■ PGD model [Vitse *et al.*]

- 4-points bending test
- prediction of damage









Task 2: Enrichment with Physics-Augmented NNs

■ Learning constitutive laws = active topic, large recent literature (PINNs, TANNs,...)

Raissi *et al.*, Physics-informed NNs: a deep learning framework for solving forward and inverse problems involving nonlinear PDEs (2019)

Huang *et al.*, Learning constitutive relations from indirect observations using deep neural networks (2020)

Hernandez *et al.*, Structure-preserving neural networks (2021)

Masi *et al.*, Thermodynamics-based ANNs for constitutive modeling (2021)

Asad *et al.*, A mechanics-informed ANN approach in data-driven constitutive modeling (2022)

Klein *et al.*, Polyconvex anisotropic hyper elasticity with neural networks (2022)

Thakolkaran *et al.*, NN-EUCLID: deep learning hyper elasticity without stress data (2022)

Linden *et al.*, Neural networks meet hyper elasticity: a guide to enforcing physics (2023)

Physics-guided architecture

- Intermediate layers can be interpreted as physical quantities that can be used in the loss function.
- Possible to impose properties such as convexity, invariance...
- **A thermodynamics-consistent architecture**

Physics-guided loss function

$$\mathcal{L} = \mathcal{L}_{\text{data}} + \mathcal{L}_{\text{phy}}$$

- “PINN” : Physics informed neural network (Raissi 2019)
- **The modified constitutive relation error**

Physics-guided initialization

- Amount of data needed is a problem : it is possible to first train the network with synthetic data generated with a physical a priori, close to the “true solution”, before training with experimental data
- **Initialization with linear isotropic law**

➡ The mCRE framework naturally integrates all recent trends

➡ No need for strain-stress (or strain-free energy) database

Task 2: Enrichment with Physics-Augmented NNs

■ General strategy (NN-mCRE)



- Generalized standard materials [Halphen & Nguyen 1975]

$$\mathbf{s} = \frac{\partial \psi}{\partial \mathbf{e}_e} ;$$

state
equations

$$\mathbf{s} = \frac{\partial \varphi}{\partial \dot{\mathbf{e}}_p}$$

evolution
laws

$$\mathcal{E}_{CRE|t}^2 = \int_{\Omega} \eta_{\psi}(\hat{\mathbf{e}}_e, \hat{\mathbf{s}}) + \int_0^t \int_{\Omega} \eta_{\varphi}(\dot{\hat{\mathbf{e}}}_p, \hat{\mathbf{s}})$$

with

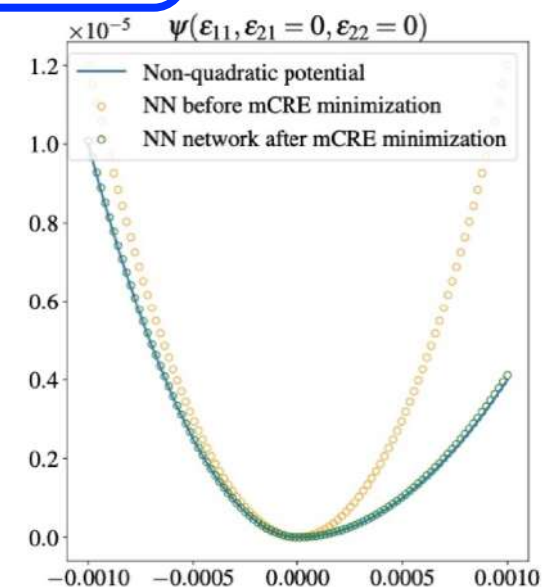
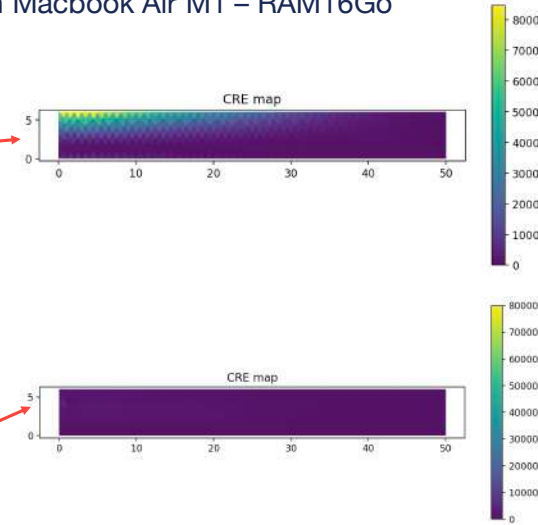
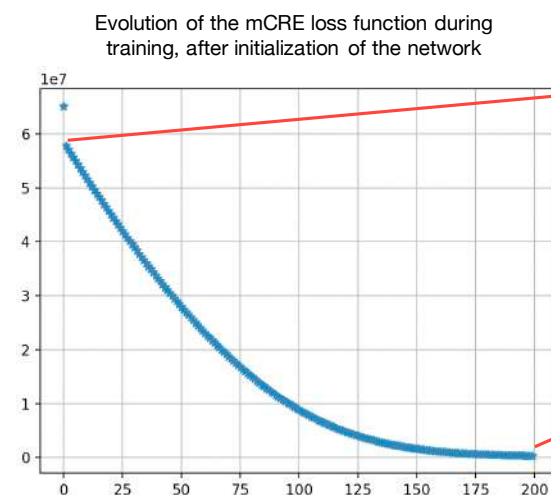
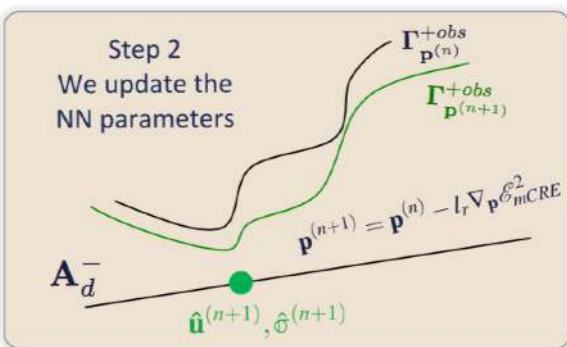
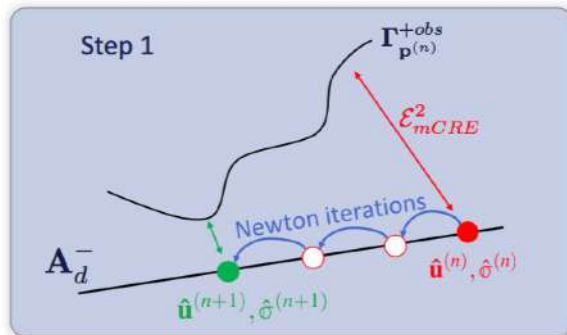
$$\begin{cases} \eta_{\psi}(\hat{\mathbf{e}}_e, \hat{\mathbf{s}}) = \psi(\hat{\mathbf{e}}_e) + \psi^*(\hat{\mathbf{s}}) - \hat{\mathbf{s}} \cdot \hat{\mathbf{e}}_e \\ \eta_{\varphi}(\dot{\hat{\mathbf{e}}}_p, \hat{\mathbf{s}}) = \varphi(\dot{\hat{\mathbf{e}}}_p) + \varphi^*(\hat{\mathbf{s}}) - \hat{\mathbf{s}} \cdot \dot{\hat{\mathbf{e}}}_p \end{cases}$$

- unsupervised strategy to learn potentials (+ internal variables) and correct model bias
- thermodynamics-consistent architecture (ICNNs [Amos 2016]), with mCRE loss function

► Truth: $\psi(\epsilon) = \frac{1}{2} E_1 (1 - d_1) \langle \epsilon_{11} \rangle_+^2 + \frac{1}{2} E_1 \langle \epsilon_{11} \rangle_-^2 + \frac{1}{2} E_2 \epsilon_{22}^2 + G \epsilon_{21}^2$

► Initial guess: $\psi(\epsilon) = \frac{1}{2} E_1 \epsilon_{11}^2 + \frac{1}{2} E_2 \epsilon_{22}^2 + G \epsilon_{21}^2$

- Implementation with Python and Pytorch. 15 minutes on Macbook Air M1 – RAM16Go

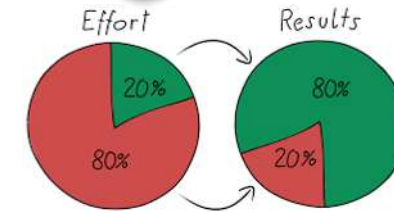


A. Benady, E. Baranger, L. C., NN-mCRE: A modified Constitutive Relation Error framework for unsupervised learning of nonlinear state laws with physics-augmented Neural Networks, *IJNME*, 125(8):e7439 (2024)

A. Benady, E. Baranger, L. C., Unsupervised learning of history-dependent constitutive material laws with thermodynamically-consistent neural networks in the modified Constitutive Relation Error framework, *CMAME*, 425:116967 (2024)

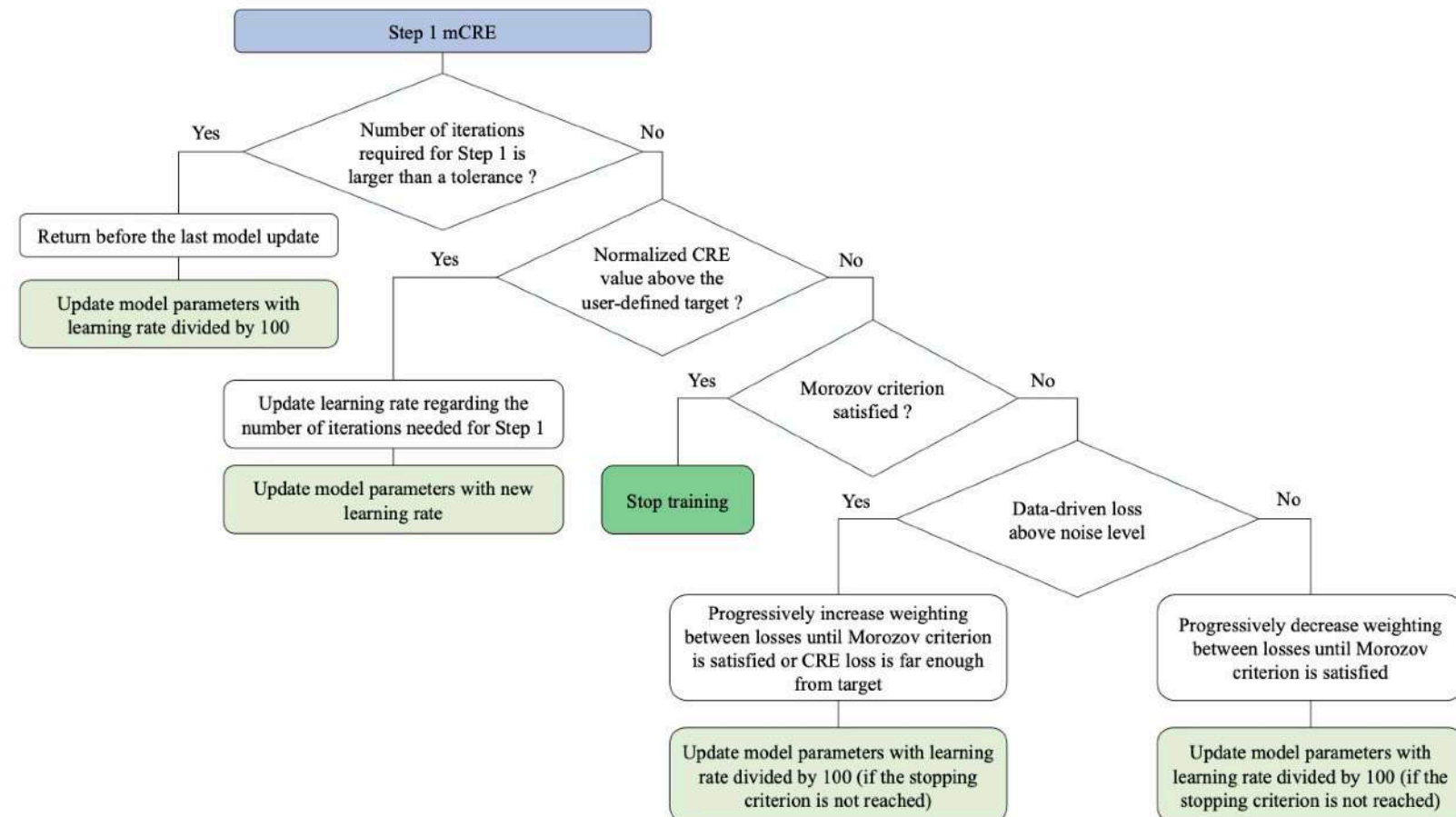
Task 2: Enrichment with Physics-Augmented NNs

■ Automatic adaptive tuning of hyper-parameters



Parameters	Strategy
learning rate	related to the number of iterations at Step 1
weighting in the loss	set using the Morozov principle (related to noise)
number of epochs	stop criteria based on the CRE modeling error term
NN initialization	trained from simple potential (e.g., linear isotropic elasticity)
batch size	fixed by the mesh (one batch is the whole structure)

➔ parameters updated during the training

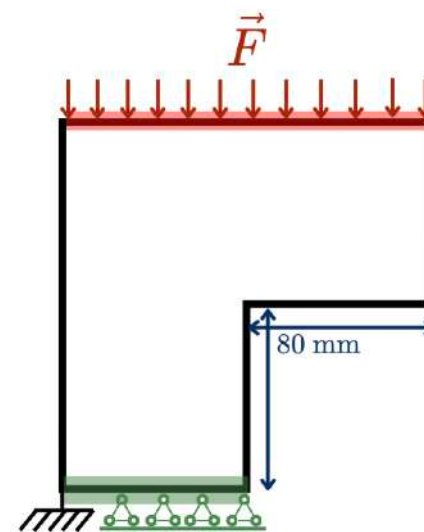
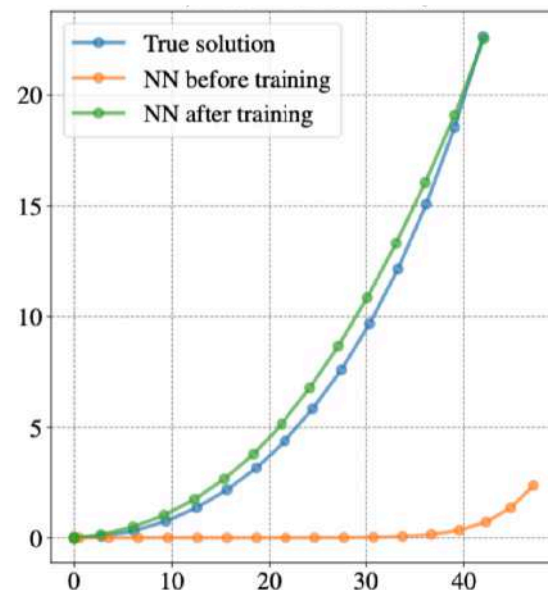
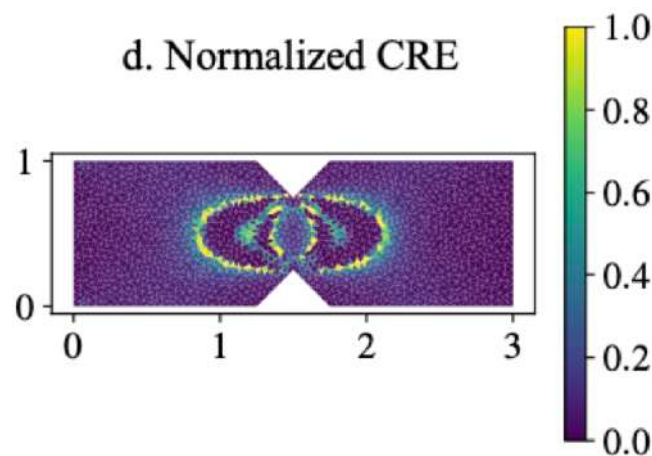
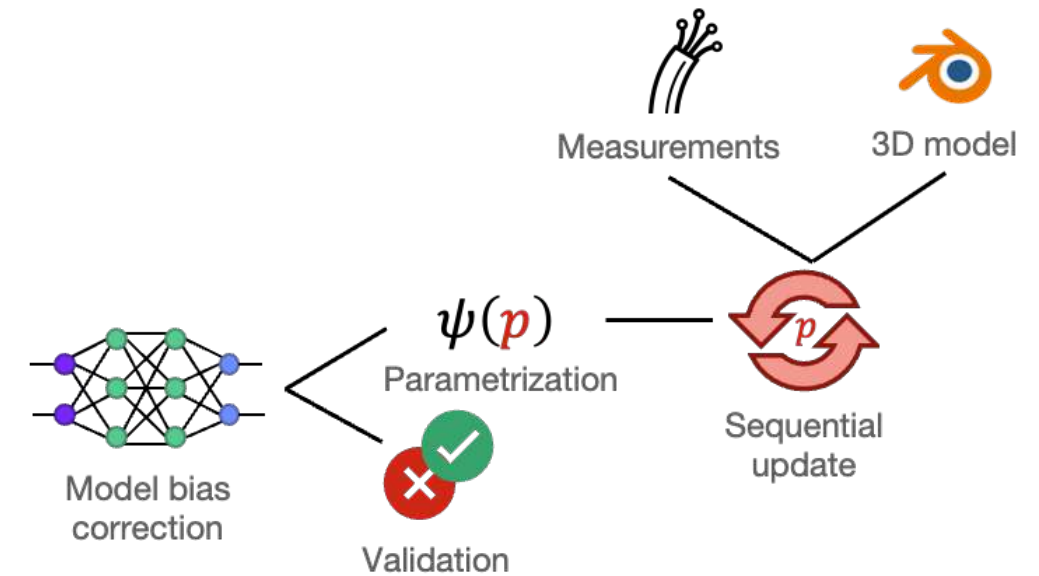


Task 2: Enrichment with Physics-Augmented NNs

■ Connexion with MDKF

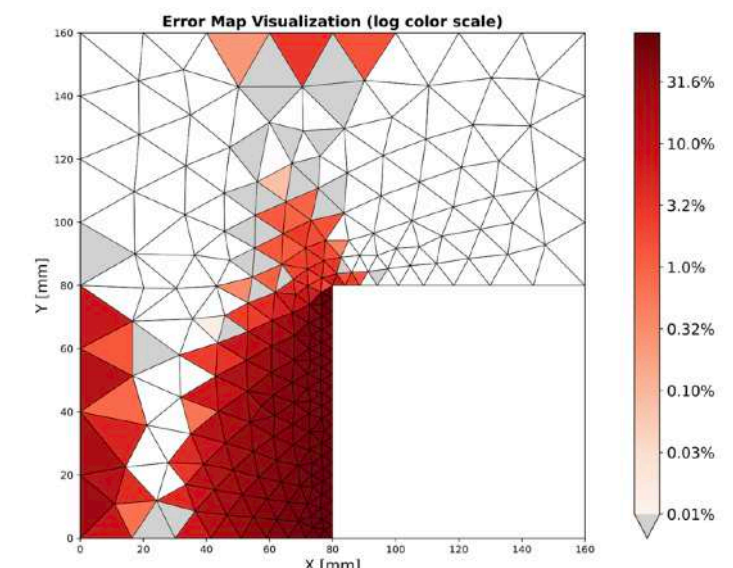
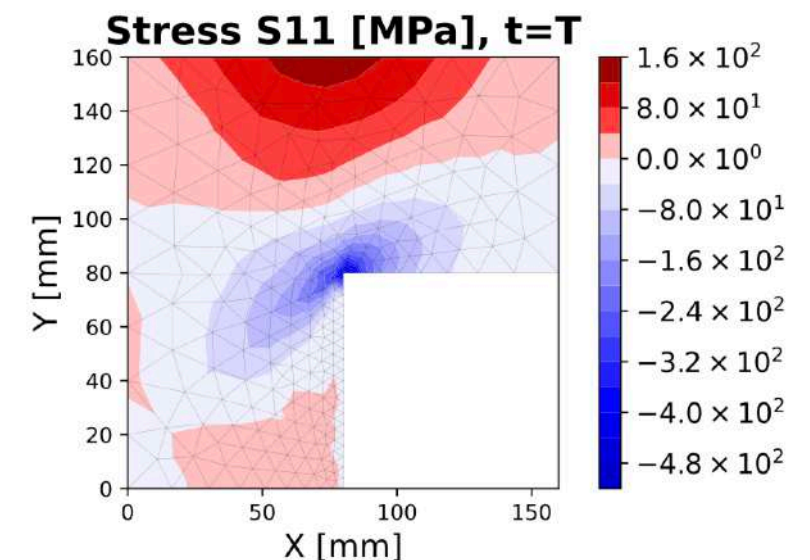
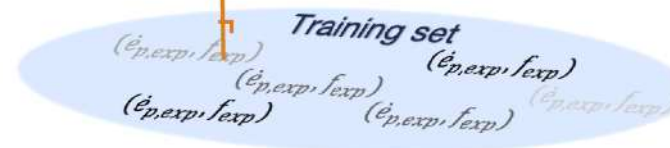
- explicit, low-parameter expressions of (ψ, φ)
- symbolic regression* [Cranmer 2023]
- extreme sparsification* [Fuhg et al. 2024]

■ Model validation



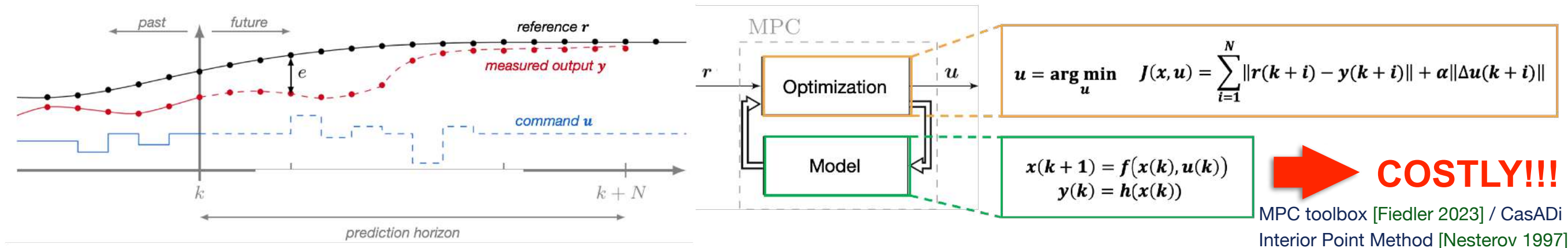
New computation on a new structure

$$\begin{aligned} \bar{\dot{e}}_p &= (\dot{e}_p, -\dot{p}) \\ \bar{f} &= (\bar{\sigma}, \bar{R}) \end{aligned}$$



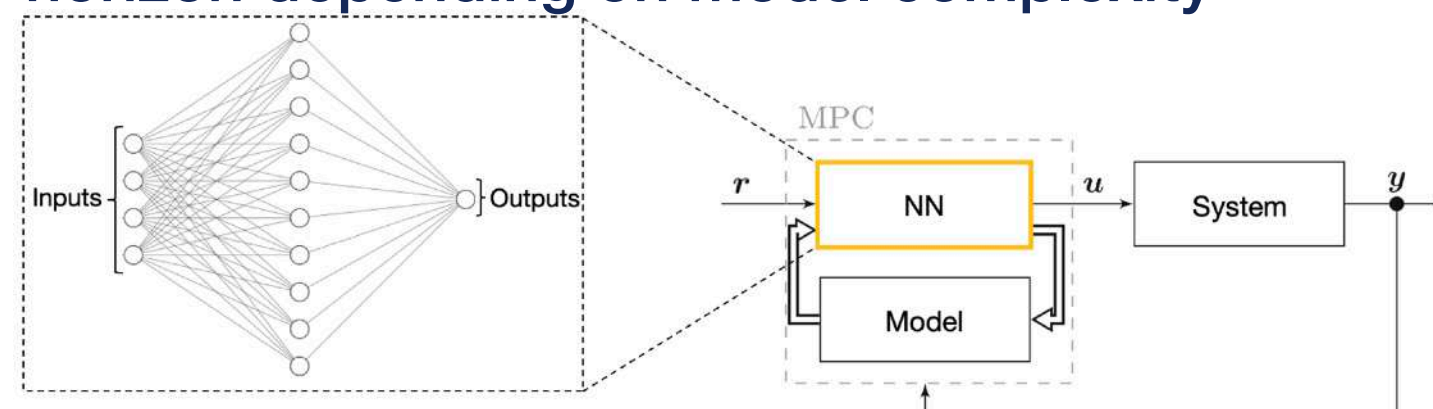
Task 3: Guaranteed Online Command Synthesis

- Use of Model Predictive Control (MPC) for integrated feedback with evolutive systems
- Solution of an optimization problem at each control time (with constraints)
- State given by optimal admissible fields of mCRE (MDKF) + UQ



- Data-driven MPC with underparametrized NNs (for real-time, high dimension,...)
 - AI used to mimic MPC optimization (policy) and quickly compute the command
 - supervised learning from MPC simulation / unsupervised learning
 - guaranteed convergence for the training error (using NTK matrix)
 - online adaptation of the horizon depending on model complexity

1 hidden layer (128 neurons)
Trained with Adam (3000 epochs)



D. Martin-Xavier, L. C., L. Fribourg, Training and generalization errors for underparametrized neural networks, *IEEE Control Systems Letters*, 7:3926–3931 (2023)

D. Martin-Xavier, L. C., L. Fribourg, MPC surrogates based on supervised or unsupervised learning for the real-time control of nonlinear systems, *IFAC Journal of Systems and Control*, 36:100409 (2026)

Task 3: Guaranteed Online Command Synthesis

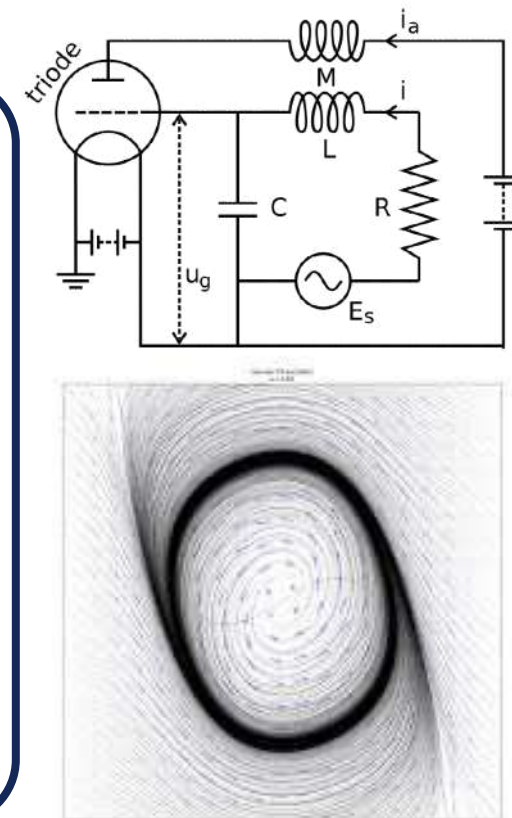
■ Application: Van der Pol oscillator

- **Model:**
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (1 - x_1^2)x_2 - x_1 - u \end{cases}$$
 models oscillations of triodes in electrical circuits

where x_1 is the **position**, x_1^{ref} the **reference**, x_2 the **speed** and u the **command**.

- **Constraints :** $u \in [-1, 1]$ and $x_1, x_2 \in [-3, 3]$

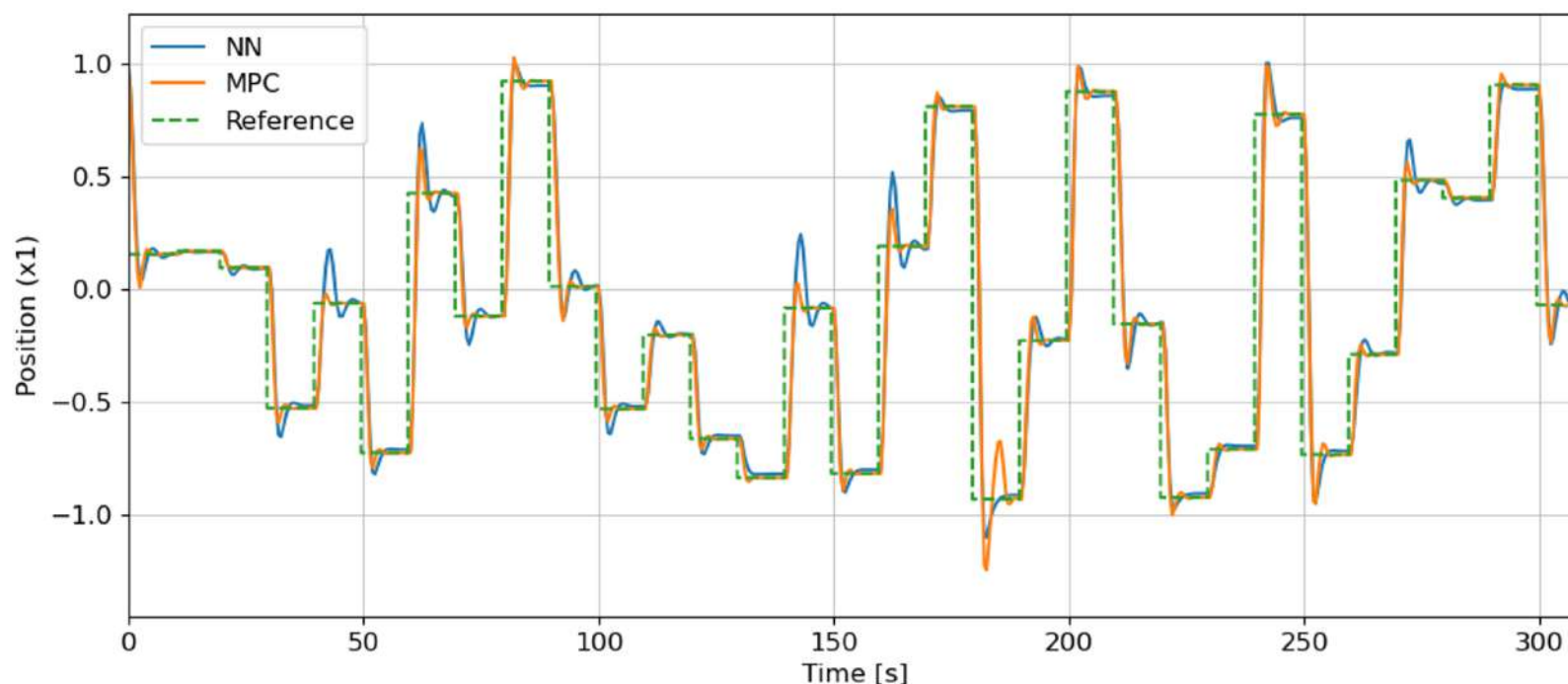
- **Goal:** make the system converge towards a reference trajectory x_1^{ref} .



prediction horizon: $N = 5$
time step: $T_S = 0.5s$

initial condition: $\mathbf{x}_0 = [1, 0]$
scaling factor: $\alpha = 0.1$

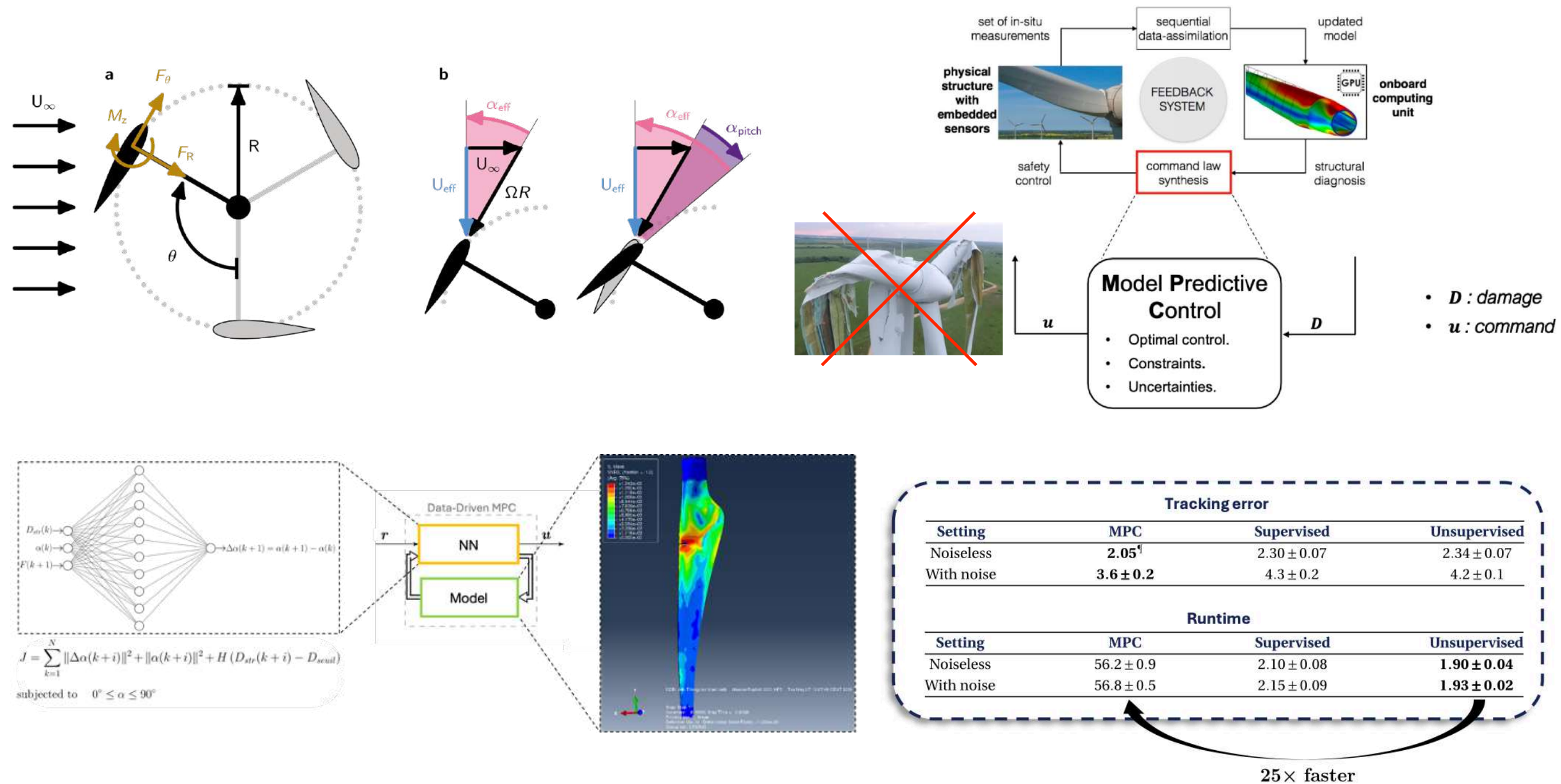
training set: $S_{train} = 6000$
test set: $S_{test} = 2000$



	MSE	CPU
MPC	0.066	2.34 ms
NN	0.067	0.17 ms

Task 3: Guaranteed Online Command Synthesis

■ Application: blade pitch control to manage damage evolution



- multiple outputs and long horizons ($N = 80$)
- stochastic disturbances (wind gusts)

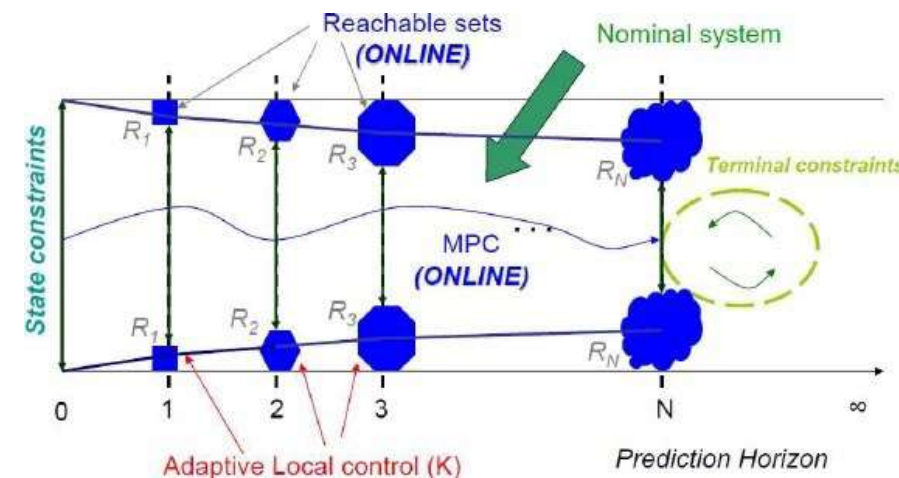
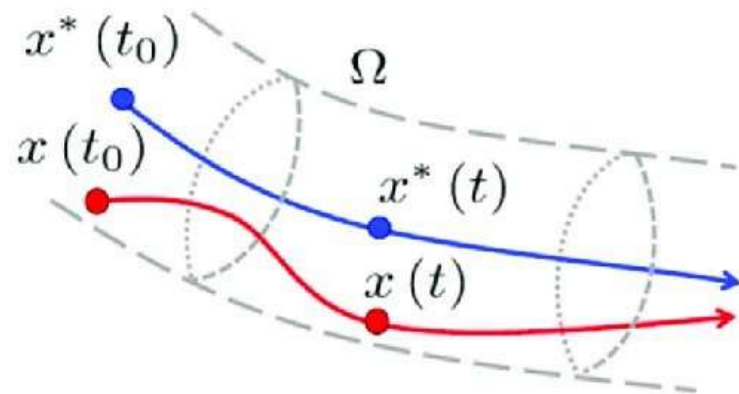
UQ with Tube MPC - Basics

Objective: robustness of the control (ensuring hard constraints) w.r.t. additional uncertainties
model bias, disturbances/noise, parametric uncertainty,...(stochastic variations)

$$\dot{\mathbf{x}}(t) = F(\mathbf{x}(t), \mathbf{u}(t)) + \mathbf{w}(t)$$

$$\mathbf{x}_{t+1} = f(\mathbf{x}_t, \mathbf{u}_t) + \mathbf{w}_t$$

- computation of a sequence of state space regions (reachability tube) [Mayne et al. 2009]
region around the nominal prediction that contains the system state under any uncertainty
calculation at each step within the prediction horizon (set of reachable sets)
- maintain all possible trajectories of the uncertain system close to reference



Combination of controllers: nominal MPC controller + ancillary feedback controller

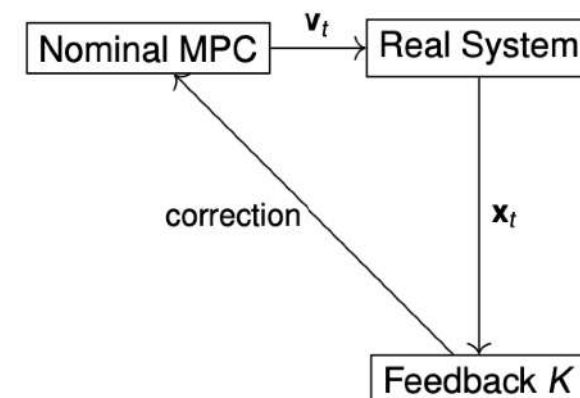
$$\mathbf{u}_t = \mathbf{v}_t + \mathbf{K}(\mathbf{x}_t - \mathbf{z}_t)$$

nominal control from disturbance-free system

real trajectory

nominal trajectory

feedback obtained from a Ricatti equation (unchanged with NN)



UQ with Tube MPC - Analysis

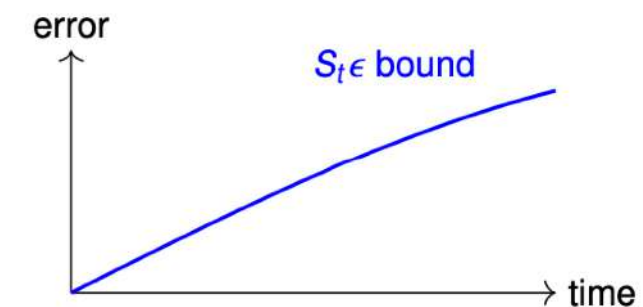
[Sivalingam *et al.* 2026]

➡ Analysis of the effect of NN approximation on the structure & behavior of tube MPC

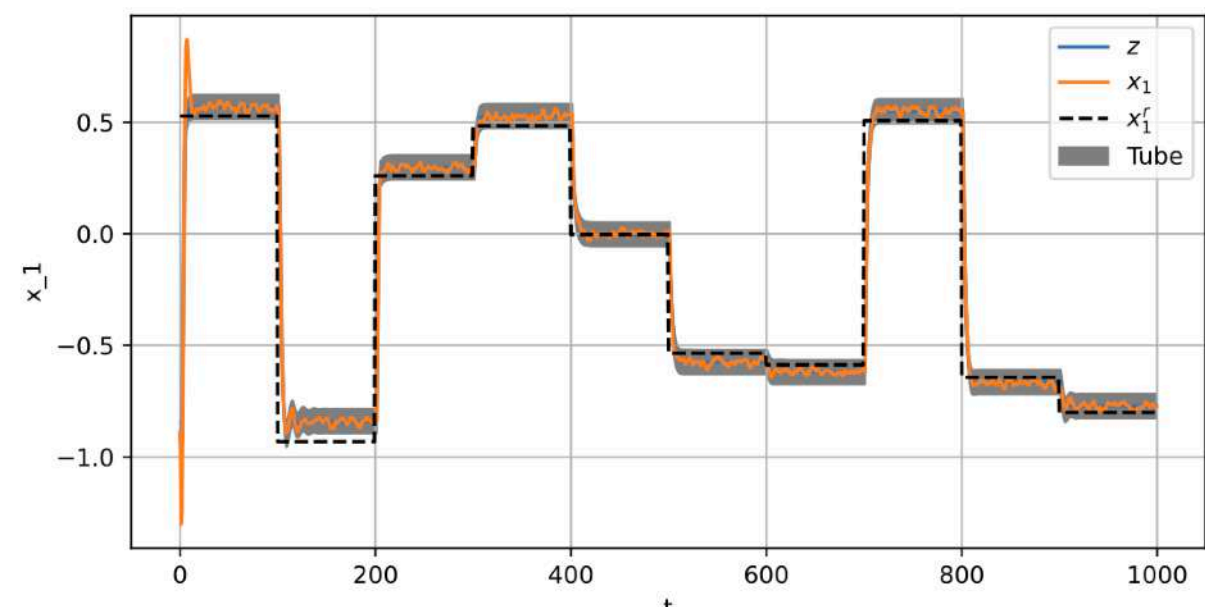
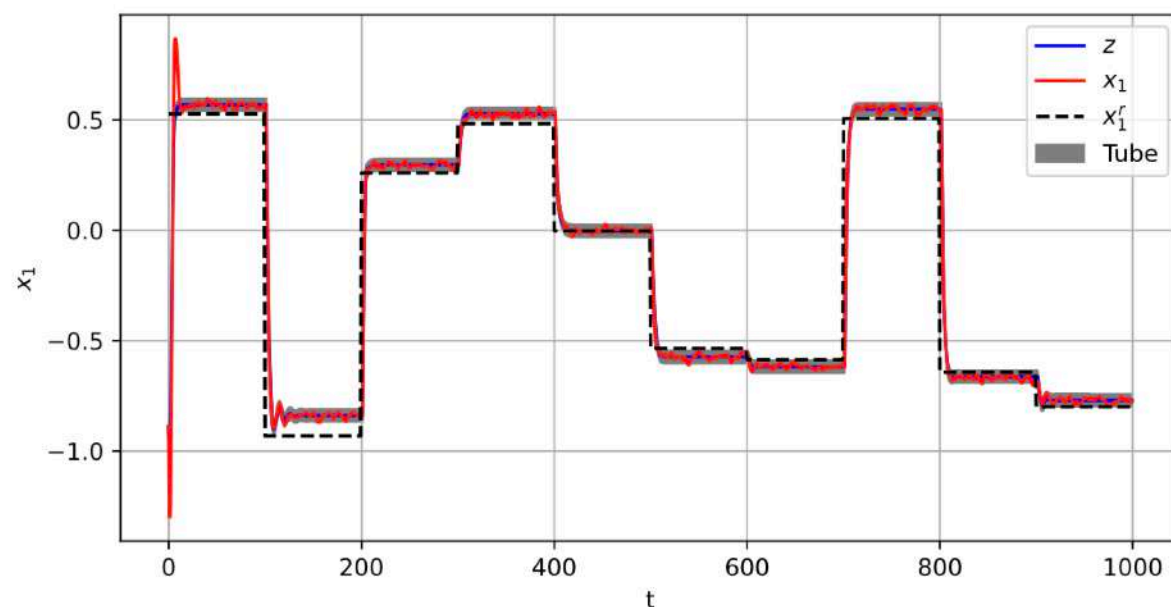
- Takes ideas from statistical ML and contraction theory (contractive dynamics systems)
- Approximation error considered as another perturbation on the control
→ *enlarged tube*
- Bound on the worst-case error in terms of the NN generalization error

$$\begin{aligned} \delta_t &= \mathbf{z}_{NN,t} - \mathbf{z}_t & \|\mathbf{z}_{NN,t} - \mathbf{x}_t^r\| &\leq S_t \epsilon + e_t^r \\ \|\delta_{t+1}\| &\leq L_z \|\delta_t\| + L_v \epsilon \\ \|\delta_t\| &\leq S_t \epsilon \text{ (with inflation factor)} \end{aligned}$$

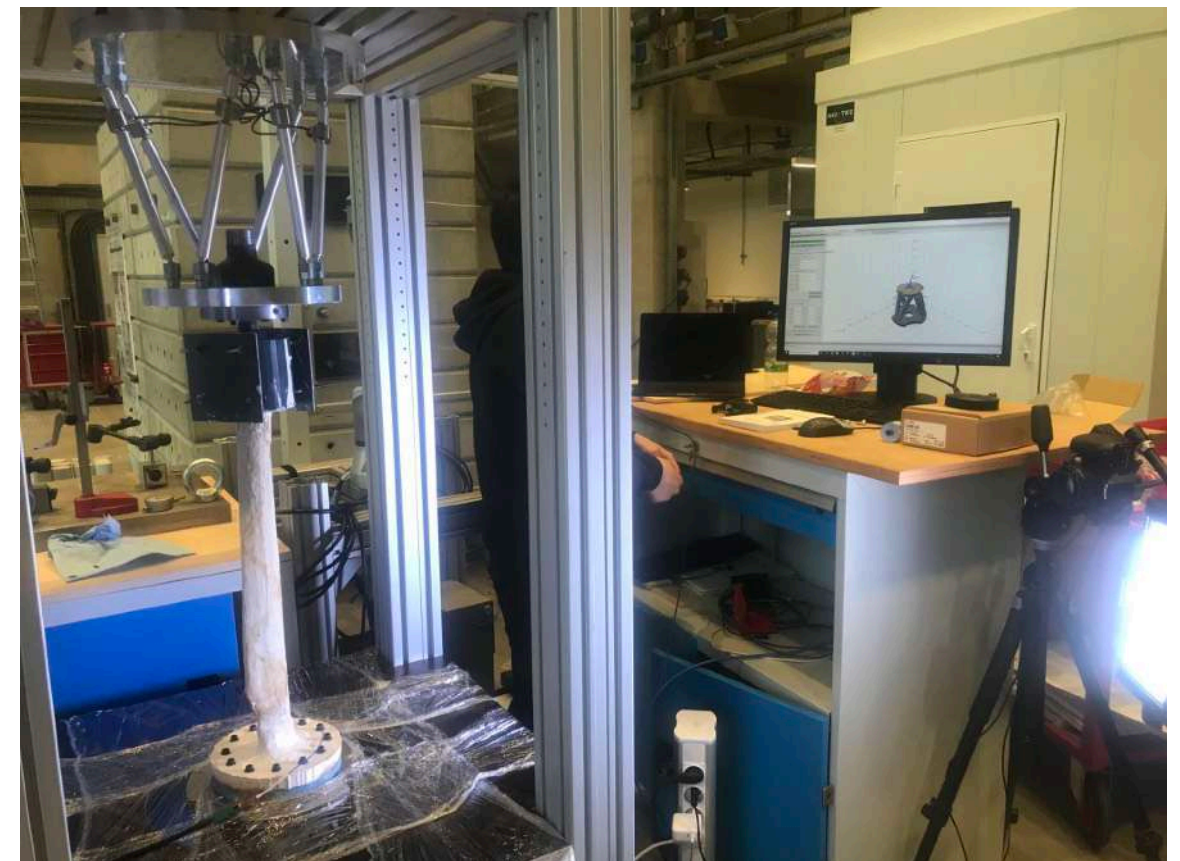
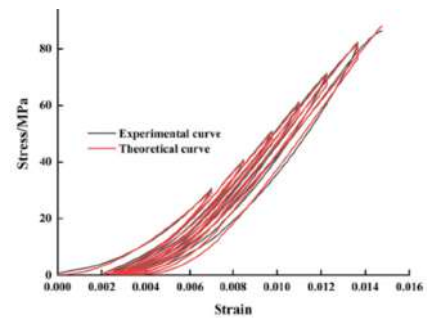
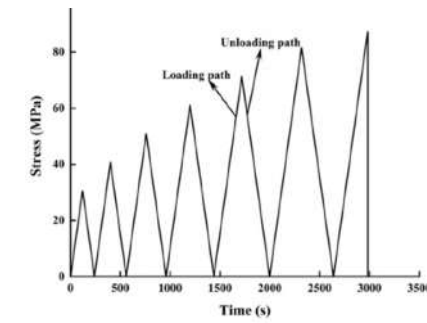
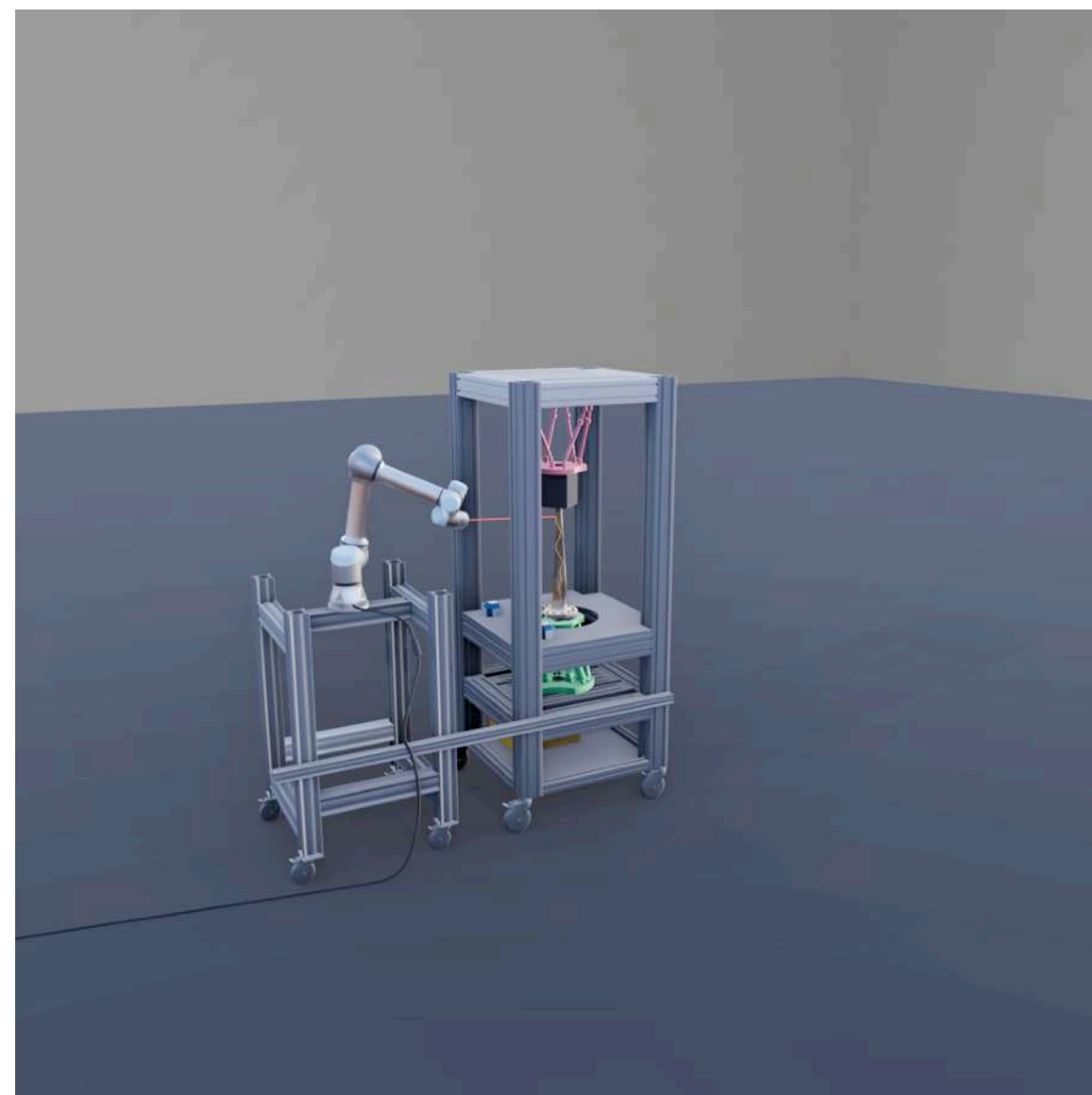
↗ NN error
↖ MPC tracking error



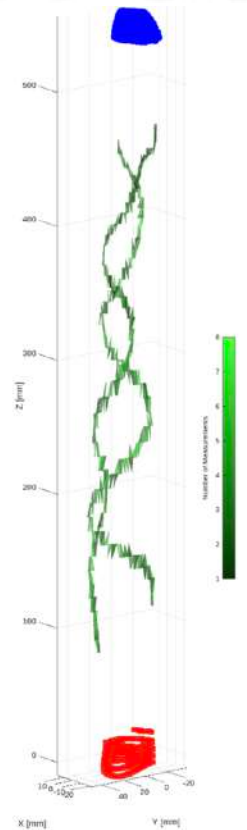
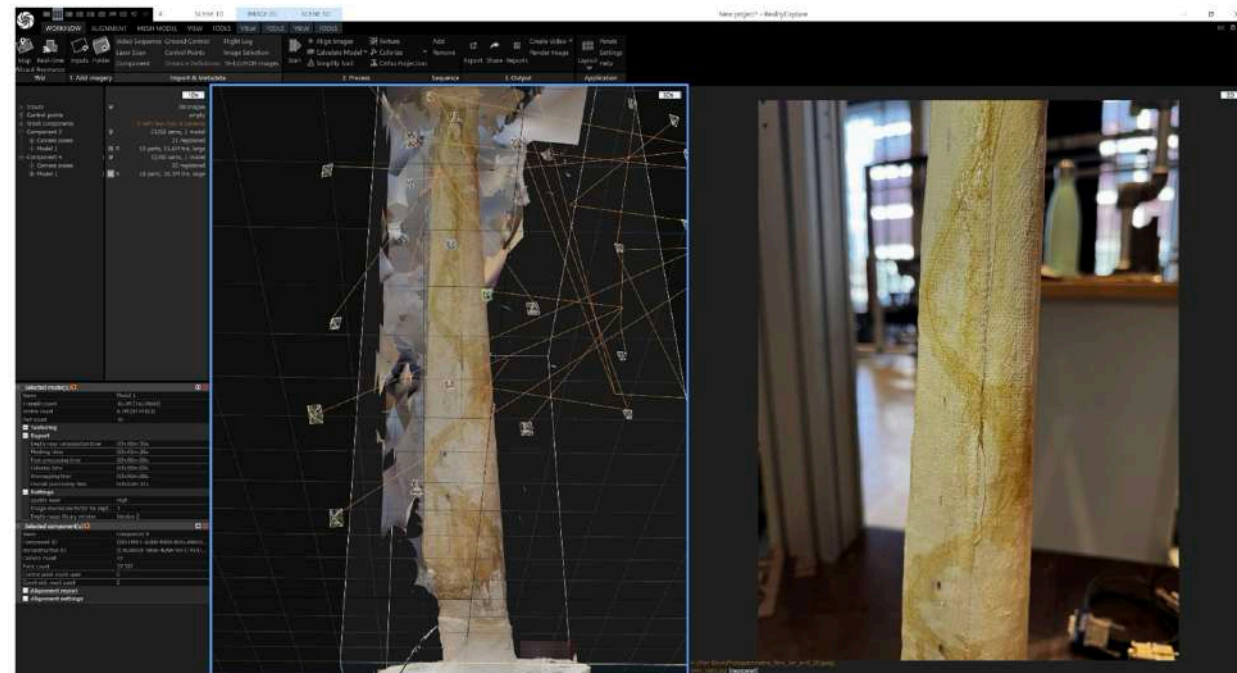
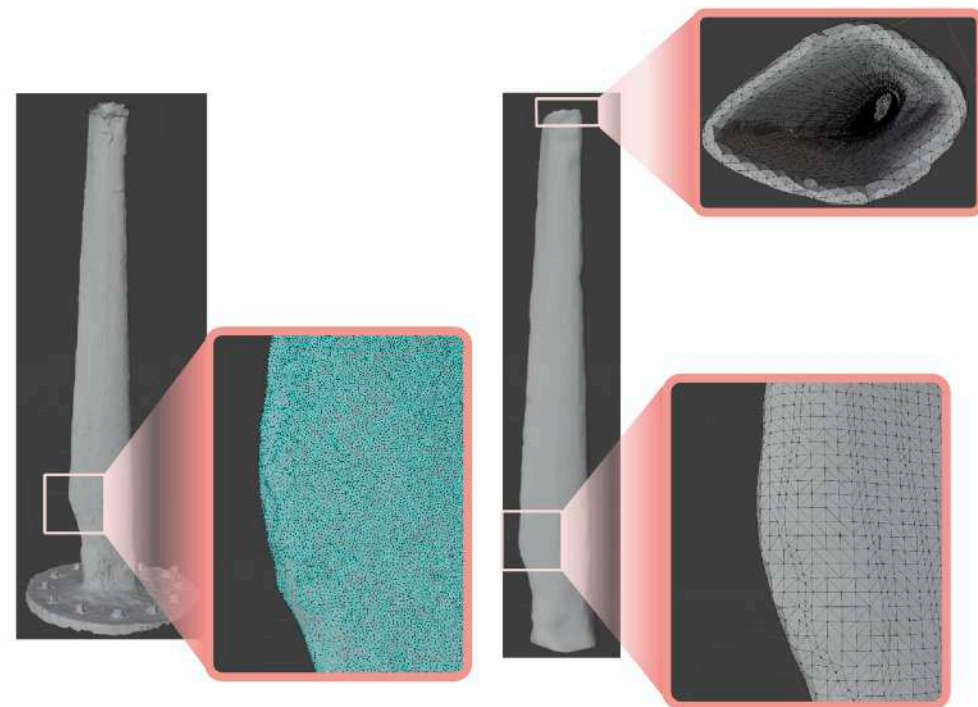
Van der Pol oscillator with/without external disturbances



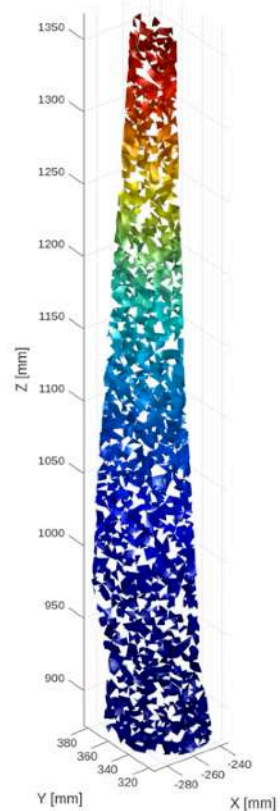
First Version of the POC



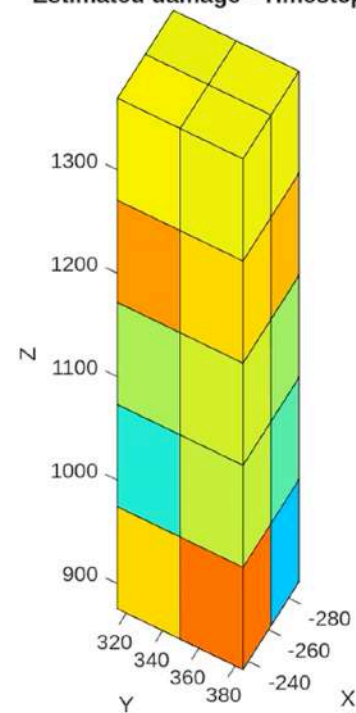
First Version of the POC



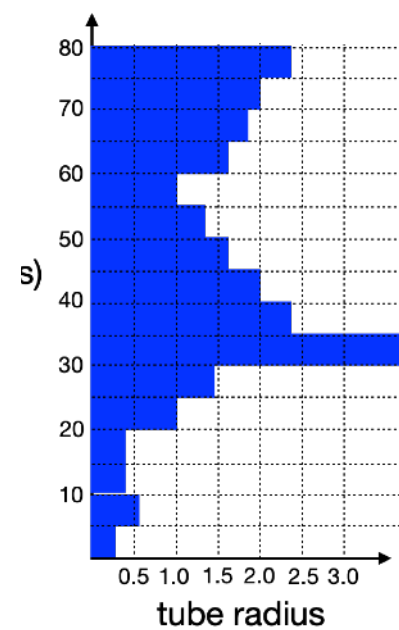
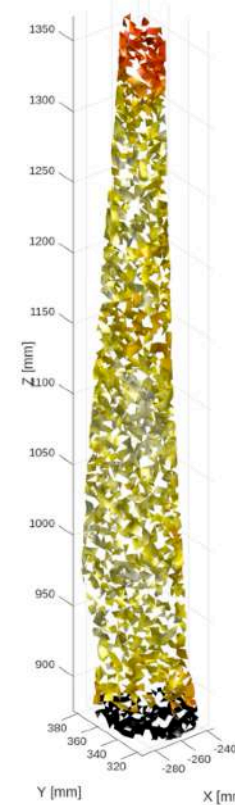
Displacement Field (Load Case 1) (Time Step 1, Scale: 100.0x)



Estimated damage - Timestep 1



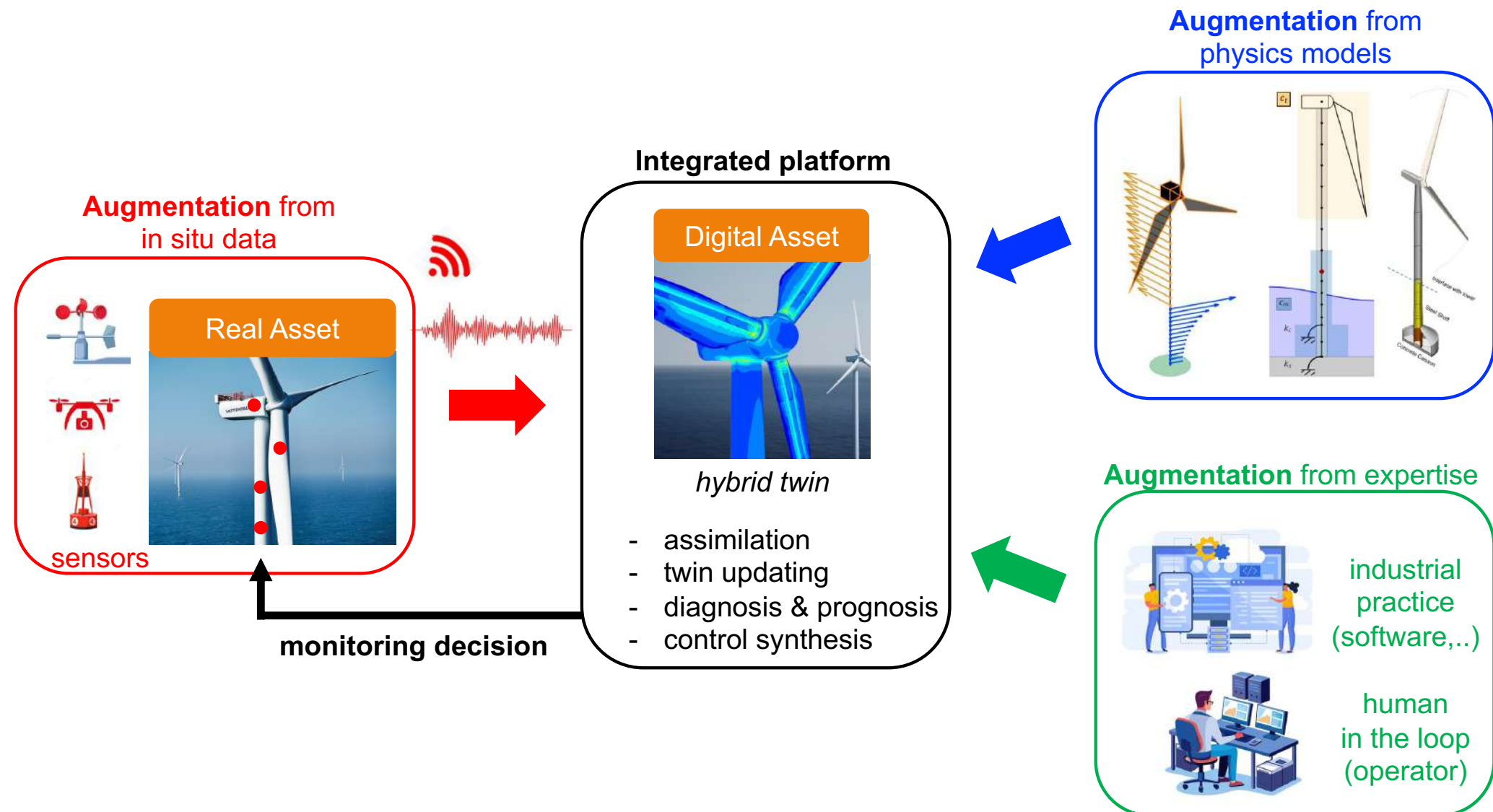
mCRE Error Distribution (Load Case 1) (Time Step 1)



What Is Next?

INCREASER (ERC-PoC, 2026-2028)

Integrated Computational Platform for Efficient Real-time Structural Monitoring and Decision Support from Augmented Digital Twinning



- hybrid twin-based monitoring and predictive maintenance for complex structures
- development of a multifunctional platform that integrates/implements ERC tools
- collaboration with industry, for evaluation and further commercial applications

Some References

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- **L. C.**, *Data assimilation and integration into computational mechanics models*, in Mathematical and Computational Modelling Across the Scales, Special Issue from the SEMA-SMAI Jacques-Louis Lions School, P. Diez & M. Giacomini (eds.), Springer (2025)
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- **S. Farahbakhsh, L. C., M. Poncelet**, Continuous structural health monitoring with a modified dual Kalman filter applied on optic fiber sensing data, *Computers & Structures* (2026)
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- **D. Martin-Xavier, L. C., L. Fribourg**, MPC surrogates based on supervised or unsupervised learning for the real-time control of nonlinear systems, *IFAC Journal of Systems and Control*, 36:100409 (2026)